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PRELIMINARY PWI SPECIFICATIONS AND THREAT LOGIC

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16. Abstract

The present Contract has as its primary objective the estimation of the potential benefit to be derived by the various users of the air-space through the implementation of Pilot Warning Instrument (PWI) systems of various degrees of sophistication. It is anticipated that the effectiveness of any PWI will depend strongly on the following: (a) the rate of alarms it generates, (b) the percentage of these alarms for which pilots detect corresponding targets, and (c) whether or not the pilots consider these targets to be dangerous. It is proposed to measure the potential effectiveness of various systems by exercising them through simulation with pilots who are busy with workloads appropriate to their mission.

In designing this simulator it has been necessary to consider the details of PWI system performance which it will be desirable to be able to simulate. The approach to this requirement has been to generate a set of preliminary PWI performance specifications which cover the range from very simple systems to those which approach Collision Avoidance Systems in complexity. An attempt has been made to include all PWI systems which have been publicly described in sufficient detail to make an estimation of performance possible. One purpose of the distribution of this report is to solicit comments from industry concerning the adequacy of coverage of these performance specifications.

In the simulation the threat environment will consist of both visible and invisible targets selected to reflect a certain density and distribution of traffic with respect to heading and airspeed. Each PWI can be characterized by an alarm volume consistent with its performance specifications; this alarm volume may be a statistical quantity. As the simulation proceeds targets approach and recede from the aircraft being "flown" in the simulator. A computer compares the position, range rate, and bearing rate of each target with the alarm volume and determines whether an alarm should be generated or not; if an alarm is generated the computer supplies signals for the activation of an appropriate display. The details of these calculations are given in two reports included as Appendices to this volume; the titles are: "Threat Logic and Alarm Rates in PWI and CAS Equipment, Part I and Part II".

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TABLE OF CONTENTS

III THREAT LOGIC AND ALARM RATES IN PWI AND CAS EQUIPMENT, PART II

PREFACE

The progress in the study of Pilot Warning Instruments (PWI) on Contract No. DOT-FA-70WA-2263 has been published in three separate reports. This report combines the three reports for ready reference in the following order:

I. Preliminary PWI Specifications, December 1971 by J. F. Lyons.

II. Threat Logic and Alarm Rates in PWI and CAS Equipment, Part I, December 1970 by V. Mangulis and W. Graham.

III. Threat Logic and Alarm Rates in PWI and CAS Equipment, Part II, May 1, 1971 by V. Mangulis and W. Graham.

Original pagination of the reports has been retained.

Preliminary PWI Specifications

J. F. Lyons

Report No. CDC-JL-1

Contract No. DOT-FA-70WA-2263

Submitted to:

Federal Aviation Administration

Department of Transportation

Submitted by:

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December 1971

SECTION I
INTRODUCTION

1.1 BACKGROUND

Under FAA Contract Number DOT FA70WA-2263, Control Data Corporation is currently studying the PWI** problem with the objectives of developing a family of definitive PWI performance specifications, and assessing the effectiveness of each hypothetical PWI in a range of threat environments. Underlying these program objectives are the recently published results of Graham and Orr* which indicate that the failure of the "see and avoid" doctrine can be attributed almost solely to the failure to see. Consistent with this result, the introduction of a PWI to assist the pilot in "seeing" could lead to an order of magnitude reduction in the probability of mid-air collisions. Here, we use PWI according to the accepted definition, which excludes the computation, indication, and command of avoidance maneuvers.

The planned program for the specification and assessment of a family of PWI's has been sub-divided into four major tasks; (a) development of PWI specifications for simulation and industry review, (b) extensive simulation experiments, (c) definition of traffic models and threat environments, and (d) an extensive program for the preparation, distribution, collection, and analysis of pilot and tower (personnel) questionnaires.

1.2 RATIONALE BEHIND GENERATION OF PWI SPECIFICATIONS

We have made a primary classification of PWI systems in accordance with derived data; i.e., the measured (or communi-

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*W. Graham and R. H. Orr "Separation of Air Traffic by Visual Means: An Estimate of the Effectiveness of the See-and-Avoid Doctrine," Proceedings of the IEEE, Vol. 58, No. 3, March, 1970.
**Pilot Warning Instrument.

cated) relative positional data which a PWI system provides at its output, prior to display. Secondary classifications are based upon: (a) the spatial coverage provided in each aircraft of a PWI pair, (b) the wavelength at which the PWI is designed to operate, and (c) major system and equipment performance features. Operating wavelengths assume considerable importance when one considers the effects of wavelength on propagation and background noise. In a like manner, one cannot properly assess the effectiveness of a hypothetical PWI without defining specific design features such as sampling rates, false alarm rates and the capacity to resolve multiple targets.

Within each of these four broad classifications there are sub-classifications and a range of parameter values. Clearly, there are practical constraints that limit the scope of the simulation to but a small fraction of the possible combinations. In this document, we present the rationale for the selection of particular parameter values, and for the elimination of those combinations which are of secondary interest.

Several factors have influenced the form and content of these preliminary PWI specifications and these should be kept in mind. First, we have attempted to represent the probable functional performance of PWI systems based on various proposed principles but we have not attempted to faithfully represent any particular PWI system proposed by a particular manufacturer. Second, we have been optimistic about the characteristics and tolerances of hardware components on the assumption that the market for a successful PWI system will warrant the required development. Third, we have assumed that suitable displays will be provided for each system to be tested by simulation. Fourth, we are trying to assess only the benefit to be derived by implementing various PWI systems; we are not concerned here with the cost of these systems and the subsequent cost/benefit analysis.

In Section II, we present and discuss the parameters and parameter values which have been considered in the classification and specification of PWI systems. In Section III, preliminary specifications are presented for hypothetical PWI systems in terms of the parameters and parameter values presented in Section II. Section IV presents a brief discussion of the planned design of the simulation experiments, with some elaboration on propagation losses, background noise, and probabilities of detection as a function of range, altitude, and the environment.

SECTION II

PWI SYSTEMS: PERFORMANCE PARAMETERS, PARAMETER VALUES, AND CLASSIFICATION

2.1 GENERAL

Tables 1 through 4 present the performance parameters which are being employed in the classification of hypothetical PWI systems. A primary classification of PWI systems is based on derived data; i.e., the positional and rate of change of position data which are derived by the system and made available as an output for display. Table 1 lists the derived data parameters being considered and the selection of parameter values for simulation experiments. Secondary PWI system classifications are based on spatial antenna coverage, operating wavelength, and system and hardware design characteristics; the selected parameters and parameter values for these secondary classifications are presented in Tables 2 through 4, respectively.

The following paragraphs present the rationale for the selection of particular parameters and parameter values. In Section III we indicate the parameters and parameter values which we have selected for preliminary PWI specifications. Section IV presents background data on the simulation experiments, detection characteristics, and propagation losses.

2.2 DERIVED DATA

Consistent with Table 1, the parameters employed in this primary classification are range, range-rate, bearing, bearing-rate, altitude difference and elevation angle. Range-rate and bearing-rate have been included to allow for additional filtering of output data based on hazard evaluation.

2.2.1 Range Data

Every PWI system derives range data in the sense that there is a characteristic probability of detection which approaches unity at some close range and approaches zero at some more distant range. Many important PWI systems depend upon this detection pro-

TABLE 1

PRIMARY CLASSIFICATION OF PWI SYSTEMS ACCORDING TO DERIVED DATA

RANGE	RANGE-RATE	BEARING	BEARING-RATE	ALTITUDE DIFFERENCE	ELEVATION ANGLE
A. Not measured: P_{DET} vs. R B. Precise Range Gate: $R \leq R_o$; $P_D = 1$ $R > R_o$; $P_D = 0$ C. Measured with Accuracy, ΔR: $\underline{\Delta R}$ 1 mile 1000 feet 200 feet 40 feet	A. None B. Measured or Calculated with Accuracy, $\Delta \dot{R}$: $\underline{\Delta \dot{R}}$ 100 feet/sec 20 feet/sec 4 feet/sec	A. None B. Location in a Sector of Width, $\Delta\varphi$: $\underline{\Delta\varphi}$ 90° 30° 10° C. Continuous Measurement with Resolution and Accuracy, $\Delta\varphi$: $\underline{\Delta\varphi}$ 30° 10° 5° 2° 1°	A. None B. Measured or Calculated with Accuracy, $\Delta\dot{\varphi}$: $\underline{\Delta\dot{\varphi}}$ $2\ ^\circ/sec$ $0.5\ ^\circ/sec$ $0.1\ ^\circ/sec$	A. None B. Filtering with Bandwidth, Δh: $\underline{\Delta h}$ ±1000 feet ± 500 feet ± 200 feet	A. None B. Determines if Above, Below, or Approximately the Same Altitude. C. Location in a Sector of Width, $\Delta\theta$: $\underline{\Delta\theta}$ 30° 10° 3 D. Continuous Measurement with Resolution and Accuracy, $\Delta\theta$: $\underline{\Delta\theta}$ 10° 5° 2° 1°

NOTE: (1) Special communicated data (e.g., heading) may be examined for effectiveness in special experiments.

cess for range discrimination and, in the usual sense, make no range measurement. In Table 1, this case has been indicated by a probability of detection (P_{DET}) function versus range (R).

While the simple probability of detection function can be likened to coarse and statistical range-gating, there are systems in which precise range-gating is achieved by setting limits on the round-trip propagation time. In practical systems, the difference in performance could be dramatic. Whereas substantial changes in received signal levels (due to antenna nulls, e.g.)would have little or no effect on the precise range-gate, the range corresponding to a given probability of detection in the crude system could change three-to-one for a ten-to-one change in signal level. Table 1 indicates the precise range-gate as another type of range data derivation.

In systems which measure range, we specify the PWI in terms of measurement accuracy. Four range accuracies have been selected for the simulation experiments.* The poorest accuracy, one mile, is probably of limited value. At the opposite extreme, an accuracy of 40 feet probably exceeds the maximum useful display accuracy, as well as the pilots ability to visually estimate range. Between these extremes range accuracies of 1000 feet and 200 feet have been selected.

2.2.2 Range-Rate Data

In the usual PWI concept, range-rate is seldom selected for measurement or calculation. It has been included here because it is of interest to determine the change in PWI effectiveness with this additional filtering of targets. Three measurement accuracies have been selected as representative of too poor, typical and practical, and better than useful.

2.2.3 Bearing Data

In Table 1 we have made a distinction between sector and continuous bearing measurements. In the former case, one employs a multiplicity of detection channels, each having a fixed field-of-view with respect to the airframe. In the latter case, one

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* Of course, range may not be displayed at all, though measured.

employs a rotating directional antenna or a multiplicity of non-directional antennas. Relative phase measurements suffice to extract bearing information with a plurality of antennas.

Sector widths of 90, 30, and 10 degrees have been selected for simulation experiments. With 360 degrees of azimuthal coverage, the corresponding number of sectors is 4, 12, and 36, respectively. We believe this range of sector widths is representative of practical and economical PWI system designs.

In the case of continuous bearing measurements, a range of accuracies from 30 degrees to 1 degree have been selected. We have specified the bearing resolution* to be equal to the measurement accuracy. This is one of the many compromises that must be made to limit the scope of the simulation experiments. We fully recognize that most system designs achieve bearing measurement accuracies which exceed the bearing resolution by an increasing margin with increasing signal-to-noise ratio: or, in contrast, that some system designs achieve bearing resolutions exceeding bearing measurement accuracies through the expoitation of time and frequency multiplexing.

2.2.4 Bearing-Rate-Data

As indicated in Table 1, we have selected, for simulation, bearing-rate measurement accuracies of 2, 0.5, and 0.1 degrees/second. The poorest accuracy is of little value in hazard discrimination, whereas the best selected accuracy probably exceeds that which can be practically and economically achieved.

2.2.5 Altitude Difference Data

The determination by the PWI of altitude difference, based on barometric data, has been specified by the width of the altitude filter used in the PWI. The width of the filter must make allowances for inaccuracies in the altimeters and in the telemetering of the data.

2.2.6 Elevation Angle Data

Elevation angle measurements have been specified, as

*The smallest difference in bearing angle between two targets at which they can be resolved.

with bearing angle measurements, according to type and accuracy. The discussion under bearing angle measurements also applies here. A special case with elevation angle measurements is the simple determination of above, below, or same altitude. This case arises from the use of separate topside and bottomside antennas, where near-equal signal strengths indicate the same altitude.

2.3 GROSS SPATIAL ANTENNA/LENS COVERAGE

The gross spatial coverage of the antenna/lens design is an important secondary aspect of PWI systems that we have selected as a classification parameter. The infinity of possibilities make this a particularly difficult area. For the purpose of simulation we will restrict our consideration to the 9 gross coverages indicated for both aircraft in Table 2. We believe this selection is sufficient to assess the sensitivity of PWI effectiveness to gross spatial coverage.

In Table 2, the indicated fields-of-view apply to the nominal beamwidth (3 db). An amplitude response typical of antennas at the particular operating wavelength will be assumed outside the nominal beamwidth. Representative amplitude responses might be gaussian, sin x/x, and cosine.

Fine-grain structure in antenna responses will be assessed by selecting specific simulation runs and threat environments that illuminate potential problems for the particular hypothetical system under evaluation.

2.4 OPERATING WAVELENGTH

All other specifications being equal, the operating wavelength of a PWI system can markedly influence the effectiveness of that system. Consequently, we have selected the operating wavelength as a secondary means of PWI system classification. Propagation and background noise vary widely with wavelength and both can have substantial effects on system performance. To assess these effects on PWI performance, we have characterized three significantly different regions of the spectrum; low microwave frequencies in the vicinity of 1500 MHz, millimeter wave-

TABLE 2

SECONDARY CLASSIFICATION OF PWI SYSTEMS ACCORDING TO GROSS SPATIAL COVERAGE

Aircraft No. 1*	
Bearing	Elevation
360°	± 60°
360°	± 30°
360°	± 10°
180°	± 60°
180°	± 30°
180°	± 10°
120°	± 60°
120°	± 30°
120°	± 10°
Up-looker Down-looker	

Aircraft No. 2*	
Bearing	Elevation
360°	± 60°
360°	± 30°
360°	± 10°
180°	± 60°
180°	± 30°
180°	± 10°
120°	± 60°
120°	± 30°
120°	± 10°
Up-looker Down-looker	

NOTES:

(1) The indicated coverage applies to the nominal (3db) beamwidth. A representative fall-off (gaussian, sin x/x, or cosine) is assumed at angles outside the indicated beamwidth. At IR wavelengths, the assumed fall-off will be a cosine function with zero response at 5 degrees outside the beamwidth.

(2) Fine-grain structure (interference pattern, nulls and scalloping, aircraft blockage, etc.) will be assessed by selecting specific simulation runs and threats that illustrate potential problems for the particular hypothetical system under evaluation.

* In PWI systems in which there are fully equipped and minimally equipped aircraft, Aircraft No. 1 is the former and Aircraft No. 2 the latter.

lengths in the vicinity of 55 GHz, and IR in the vicinity of 0.9 microns. Table 3 indicates the propagation and background effects to be taken into account in the simulation experiments in each case.

At low microwave frequencies, the propagation loss is limited to a $1/R^2$ spreading loss, and background effects are not significant. Furthermore, precipitation does not introduce significant atmospheric attenuation, particularly with the relatively short ranges of interest for PWI systems.

In the region of 55 GHz, propagation exhibits additional attenuation due to oxygen absorption and precipitation. Except in the case of very heavy precipitation, the oxygen absorption is the predominate source of atmospheric attenuation. Background effects in this frequency range are not sufficiently significant to warrant simulation.

In the IR region, the propagation loss is a sensitive function of the state of the atmosphere. Fortunately, the effects on visible propagation are similar to those at IR wavelengths and, knowing the visibility, one can conveniently estimate the IR attenuation. At IR wavelengths, background noise is critically important. External background noise exceeds the level of noise which is generated within IR systems and practical equipments exhibit performance which is a sensitive function of background. Background noise varies widely as bright clouds, blue sky, and the sun pass through the IR field-of-view.*

2.5 SYSTEM AND EQUIPMENT DESIGN CHARACTERISTICS

Table 4 summarizes the parameters and parameter values selected for a secondary classification of PWI systems according to design characteristics. We have attempted to focus on those characteristics which bear most heavily on PWI effectiveness. One should recall that the complete range of parameter values will not be simulated for every specified hypothetical PWI system. Section III indicates the range of parameter values

* As indicated in Table 3, the representation of background noise in the simulation will be restricted to the saturation effects in the vicinity of the sun.

TABLE 3

SECONDARY CLASSIFICATION OF PWI SYSTEMS ACCORDING TO OPERATING WAVELENGTH

PROPAGATION EFFECTS	BACKGROUND CHARACTERISTICS
A. Simple $1/R^2$ Behavior	A. Not a significant factor in overall system performance.
B. Simple $1/R^2$ + Atmospheric Attenuation	B. A significant factor in overall system performance.
C. Simple $1/R^2$ + Atmospheric Attenuation + Amplitude Scintillation	
NOTES:	**NOTES:**
(1) Atmospheric attenuation is a function of the environment and altitude, in addition to wavelength.	(1) Background characteristics will be introduced only in the representation of IR and laser systems.
(2) Scintillation effects vary with the environment and altitude.	(2) The representation of background characteristics in the simulation experiments will be restricted to a saturation effect at spatial angles within 45 degrees of the sun.

NOTE: With the simulation of radar-type PWI systems, the propagation loss varies as $1/R^4$ and one must introduce the effective scattering cross-section of targets, as well as signal fluctuation, if high-gain retrodirective reflectors are not employed. Because of these complications, the simulation of such systems will be treated as a special case.

TABLE 4

SECONDARY CLASSIFICATION OF PWI SYSTEMS ACCORDING TO DESIGN CHARACTERISTICS

TOLERANCE IN POWER BUDGET	DETECTION RANGE	MEAN FALSE ALARM INTERVAL	SAMPLING INTERVALS & PROCESSING DELAYS	MULTIPLE TARGET DISCRIMINATION
+2 db & -5 db +5 db & -10 db +7 db & -15 db +10 db & -20 db	A. Range, R_o, where P_{DET} equal 0.99: R_o 20 miles 10 miles 5 miles 3 miles 2 miles 1 mile B. Decoding Logic: Single Pulse - P_D Two Pulses - P_D^2 Three Pulses - P_D^3	t_{FA} ∞ 1 hour 10 min 2 min 30 sec 10 sec 3 sec 1 sec	A. Sampling Interval, T_s: B. Processing Delay, T_D: T_s & T_D 0.1 sec 0.3 sec 1.0 sec 3.0 sec 10.0 sec	A. None B. Per Resolution Eleme C. N Per Resolution Element D. All

ANGLE TRACKING CAPABILITY	OPERATIONAL CONTROLS
A. None B. No error with $\varphi \leq \varphi_{MAX}$: φ_{MAX} 1°/sec 3°/sec 10°/sec C. All Rates	A. Range Selector B. Threshold Level Set or Sensitivity Control

NOTES:

(1) Processing delay times exclude memory functions which are treated as a display characteristic.

(2) Susceptibility to fruiting, mutual interference, multipath problems, and similar performance deficiencies are treated as engineering problems that could be reduced, through design, to an acceptable level.

(3) The effects of no antenna stabilization, horizon stabilization, and three-axis stabilization will be evaluated in specialized simulation experiments.

(4) With IR, detection range applies to clear weather; visibility equal 64 miles.

(5) Only a small fraction of the indicated parameter values will actually be simulated.

that will be simulated for specified systems.

2.5.1 Tolerance in Power Budget

In the design of a typical system, one usually selects a combination of component characteristics and specifications that yield some nominal operating range under a set of nominal conditions. These conditions might include light rain, an average antenna gain, and nominal transmitter power. Deviations from the assumed nominal conditions (e.g., geometry corresponding to an antenna null) have the effect of increasing or decreasing the detection range. In systems which do not measure range, the possible wide variation in detection range can significantly effect PWI effectiveness. This area will be explored in the simulation through the introduction of a power budget tolerance with the parameter values indicated in Table 4.

In contrast to the nominal design approach, one often designs a system to guarantee some minimum detection range under so-called "worst-case" conditions. With this approach, one is specifying the lower limit of the tolerance in the power budget. For the simulation experiments, this case will be accomodated by selecting a nominal range so that the guaranteed range corresponds to the lower limit of the selected tolerance range.

2.5.2 Detection Range

In Table 4, we have selected nominal detection ranges from one (1) to twenty (20) miles. Further we have allowed for detection characteristics corresponding to single pulse detection and two-of-two detection* Here we are more concerned with the real-time detection characteristics, from sample-to-sample, than the cumulative probability of detection. Probability of detection characteristics are presented in Section IV.

Table 4 indicates the false alarm intervals of interest for the simulation experiments. At one extreme, there are no false alarms: at the opposite extreme, the false alarm interval equals one second. Here, the false alarm interval applies

* three consecutive detections may be required (certain IR systems)

to the average time interval between successive false alarms in any resolution element of the display. With X resolution elements in the display, the average time interval between successive false alarms in the same resolution element would be X times the value indicated in Table 4.

We anticipate that excessive false alarm rates will substantially undermine pilot confidence and be manifest in poor PWI effectiveness.*

2.5.4 Sampling Intervals and Processing Delays

Sampling intervals and processing delays have been selected as simulation parameters because inadequacy in these areas can lead to problems in multiple target resolution and [illegible] tracking. Both lead to a time lag between data derivation and data display.

As an example of a problem that one might anticipate, consider one aircraft passing tangentially to another aircraft at a range of 3000 feet and with a relative velocity of 200 knots. In this case the peak instantaneous rate of change of bearing would be about 6 degrees/second. Clearly, without special logic, those systems with a sampling rate of 1/second, two-of-two successive pulse decoders, and a bearing resolution of less than 3 degrees would have a problem. Furthermore, the example is hardly an extreme case.

In the same example, excessive processing delay could result in a display presentation that indicated target positions which were several degrees in error.

With sector location schemes, the problems of this nature that might arise are probably less severe, but there are still problems in the vicinity of sector boundaries.

As one attempts to track targets which traverse resolution elements in less than the sampling interval, there are problems that arise with multiple target discrimination.

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*Nonetheless, it is comforting to recall that a slight increase in threshold levels can increase false alarm intervals by orders of magnitude with only a nominal decrease in operating range.

2.5.5 Multiple Target Discrimination

The ability to resolve and discriminate between multiple targets will be specified according to one of four possibilities: (a) no multiple target discrimination; (b) discrimination between targets in different system resolution elements; (c) discrimination between targets in different resolution elements and among N targets in a common resolution element; and (d) resolution of all multiple targets. Table 4 reflects this classification.

2.5.6 Angle Tracking Capability

Table 4 lists a range of angle tracking capabilities that have been considered for the simulation experiments. Earlier discussions indicated the significance of this performance specification.

2.5.7 Operational Controls

For the purpose of the simulation experiments, we have made provision for the inclusion of a range selection control and a threshold level setting control. A range selector might be employed with proximity-warning systems which indicate the intrusion of another aircraft into a protective volume (e.g., sphere or a sphere truncated by altitude difference); to be useful, the range must be accurate, such as can be measured by observation of round-trip propagation delay.

A threshold level set control would be employed to reduce detection range in high traffic environments, or to reduce false alarms with increasing background noise levels. In IR systems, the problems with background noise may demand such a control until substantially brighter sources become available.

2.5.8 Other

Several footnotes to Table 4 indicate the reasoning applicable to memory, fruiting, mutual interference, multipath, and antenna or lens stabilization.

SECTION III

PRELIMINARY HYPOTHETICAL PWI SPECIFICATIONS

2.1 GENERAL

The pages of this section present preliminary specifications for seven hypothetical PWI systems. These specifications are preliminary in that they are subject to modification. Also, the list of seven selected systems is subject to additions and deletions.

We are hopeful that an industry review of these specifications will highlight the errors, shortcomings, and important omissions. In the final selection of candidate PWI's for simulation, we will attempt to focus on the most important systems within the limits of available simulation time.

2.2 SPECIFIC

The following pages present preliminary specifications, for seven PWI systems. The rationale for the selection of particular parameters and parameter values was presented in Section II.

PRELIMINARY SPECIFICATION FOR HYPOTHETICAL PWI I

A. BRIEF DESCRIPTION

PWI I detects all aircraft which intrude into a roughly spherical protective volume centered on own aircraft. This system provides an output indication that one or more aircraft are within this protective volume and whether it is above, below, or at the same altitude. Range-gating to define this volume depends solely upon a statistical probability of detection.

B. PRIMARY CLASSIFICATION ACCORDING TO DERIVED DATA

(1) Range:

(a) Not measured

(b) P_{DET} versus R

(2) Range-rate:------------------------------none

(3) Bearing:---------------------------------none

(4) Bearing-rate:----------------------------none

(5) Altitude difference:---------------------none

(6) Elevation angle:-------------------------above, below, or same altitude (referred to air-frame)

C. SPATIAL COVERAGE

(1) Aircraft number 1:

(a) Bearing:-----------------------------360^o

(b) Elevation:---------------------------$\pm 60^o$

(2) Aircraft number 2:

(a) Bearing:-----------------------------360^o

(b) Elevation:---------------------------$\pm 60^o$

D. OPERATING WAVELENGTH

In the simulation experiments we will examine the performance of this system at all three wavelengths of interest.

(1) Microwave:

(a) Propagation:-------------------------$1/R^2$

(b) Background:--------------------------NA

(2) Millimeter (55GHz):

(a) Propagation:------------------------$1/R^2 + \alpha$, where α is a function of altitude and precipitation

(b) Background:-------------------------NA

(3) IR:

(a) Propagation:------------------------$1/R^2 + \alpha$ + scintillation; α is a function of visibility, and scintillation will be taken account of in the power tolerance

(b) Background:-------------------------to be disregarded with this hypothetical PWI

E. SYSTEM AND EQUIPMENT DESIGN PARAMETERS

Simulation runs will be planned to examine the performance of this system for the indicated range of parameter values.

(1) Tolerance in power budget:----------------+5db and -10db
+10db and -20db

(2) Detection range:

(a) Range R_o, for P_{DET} equal 0.99--------10, 4, and 2 miles

(b) Decoding logic:----------------------P^2_{DET}, P^3_{DET} with IR

(3) False alarm interval:---------------------infinite

(4) Sampling intervals and processing delays:

(a) Sampling interval:------------------1 second

(b) Processing delay:-------------------none

(5) Multiple target discrimination:-----------none

(6) Angle tracking capability:----------------NA

(7) Operational controls:---------------------Sensitivity control with IR

PRELIMINARY SPECIFICATION FOR HYPOTHETICAL PWI II

A. BRIEF DESCRIPTION

PWI II detects all aircraft which intrude into a precisely defined spherical protective volume centered on own aircraft. This system indicates the presence of an intruder within a selected range and further indicates whether the intruder is above, below, or at the same altitude. Range-gating is based upon an observation of round-trip propagation time, and the system does not indicate multiple targets. The system specification is idealistic in the sense that performance is guaranteed throughout the full range of tolerance levels.

B. PRIMARY CLASSIFICATION ACCORDING TO DERIVED DATA

(1) Range:

(a) Precise range-gate

(b) $R \leq R_0$; $P_{DET} = 1$ and $R > R_0$; $P_{DET} = 0$

(2) Range-rate:--------------------------------none

(3) Bearing:-----------------------------------none

(4) Bearing-rate:------------------------------none

(5) Altitude difference:-----------------------none

(6) Elevation angle:---------------------------above, below, or same altitude (referred to airframe)

C. SPATIAL COVERAGE

(1) Aircraft number 1:

(a) Bearing:----------------------------360^o

(b) Elevation:--------------------------$\pm 60^o$

(2) Aircraft number 2:

(a) Bearing:----------------------------360^o

(b) Elevation:--------------------------$\pm 60^o$

D. OPERATING WAVELENGTH

With the system as specified, the differences in propagation and background noise will have no effects on PWI perfor-

mance. Consequently, to the extent that it is practical to meet the indicated specifications, the results of the simulation are applicable to all wavelengths.

E. SYSTEM AND EQUIPMENT DESIGN PARAMETERS

Simulation runs will be planned to examine the performance of this system for the indicated range of parameter values.

(1) Tolerance in power budget:---------------NA

(2) Detection range:-------------------------selected by operator

(3) False alarm interval:--------------------infinite

(4) Sampling intervals and processing delay:

(a) Sampling interval:------------------1 second

(b) Processing delay:-------------------none

(5) Multiple target discrimination:-----------none

(6) Angle tracking capability:---------------NA

(7) Operational controls:

(a) Range selector:---------------------1, 2, 3, miles

PRELIMINARY SPECIFICATION FOR HYPOTHETICAL PWI III

A. BRIEF DESCRIPTION

PWI III detects all aircraft which intrude into a precisely-defined spherical volume which is centered on own aircraft and truncated in altitude. This system indicates the presence of intruders which are within both altitude and range proximity. A range selector is provided and range-gating is based upon an observation of round-trip propagation time. The system does not indicate multiple targets.

This system is similar to PWI II; it differs from PWI II only in that altitude filtering has been added. As in the case of PWI II, the system specification is idealistic in the sense that performance is guaranteed throughout the full range of tolerance levels.

B. PRIMARY CLASSIFICATION ACCORDING TO DERIVED DATA

(1) Range:

(a) Precise range-gate

(b) $R \leq R_o$; $P_{DET} = 1$ and $R > R_o$; $P_{DET} = 0$

(2) Range-rate:----------------------------------none

(3) Bearing:-------------------------------------none

(4) Bearing-rate:--------------------------------none

(5) Altitude difference:---------------------± 1000 feet
± 500 feet

(6) Elevation angle:-------------------------------none

C. SPATIAL COVERAGE

(1) Aircraft number 1:

(a) Bearing:-----------------------------360^o

(b) Elevation:---------------------------$\pm 60^o$

(2) Aircraft number 2:

(a) Bearing:-----------------------------360^o

(b) Elevation:---------------------------$\pm 60^o$

D. OPERATING WAVELENGTH

With the system as specified, the differences in propagation and background noise will have no effects on PWI performance. Consequently, the results of the simulation are applicable to all wavelengths, to the extent that it is practical to meet the indicated specifications.

E. SYSTEM AND EQUIPMENT DESIGN PARAMETERS

Simulation runs will be planned to examine the performance of this system for the indicated range of parameter values.

(1) Tolerance in power budget:----------------NA

(2) Detection range:--------------------------selected by operator

(3) False alarm interval:---------------------infinite

(4) Sampling intervals and processing delay:

(a) Sampling interval:------------------1 second

(b) Processing delay:-------------------none

(5) Multiple target discrimination:-----------none

(6) Angle tracking capability:----------------NA

(7) Operational controls:

(a) Range selector:---------------------1, 2, 3 miles

PRELIMINARY SPECIFICATION FOR HYPOTHETICAL PWI IV

A. BRIEF DESCRIPTION

PWI IV detects all aircraft which intrude into a roughly spherical volume centered on own aircraft. Range-gating to define this volume depends solely upon a statistical probability of detection. In addition to the detection and indication of intruders, this system measures bearing and resolves multiple targets not in the same bearing resolution element. Neither altitude difference nor elevation angle are measured; the system does, however, indicate above, below, or the same altitude.

B. PRIMARY CLASSIFICATION ACCORDING TO DERIVED DATA

(1) Range:
 (a) Not measured
 (b) P_{DET} versus R
(2) Range-rate:------------------------------none
(3) Bearing:
 (a) Sectors:----------------------------90°, 30°, and 10°
 (b) Resolution and Accuracy:------------5° and 2°
(4) Bearing rate:---------------------------none
(5) Altitude difference:---------------------none
(6) Elevation angle:-------------------------above, below, or same altitude (referred to air-frame)

C. SPATIAL COVERAGE

(1) Aircraft number 1:
 (a) Bearing:----------------------------360° and 180°
 (b) Elevation:--------------------------$\pm 30^{\circ}$ and $\pm 10^{\circ}$ for IR
(2) Aircraft number 2:
 (a) Bearing:----------------------------360°
 (b) Elevation:--------------------------$\pm 30^{\circ}$

D. OPERATING WAVELENGTH

In the simulation experiments we will examine the performance of this system at all three wavelengths of interest.

(1) Microwave:

(a) Propagation:------------------------$1/R^2$

(b) Background:-------------------------NA

(2) Millimeter (55GHz):

(a) Propagation:------------------------$1/R^2 + \alpha$, where α is a function of altitude and precipitation

(b) Background:-------------------------NA

(3) IR:

(a) Propagation:------------------------$1/R^2 + \alpha$ + scintillation; α is a function of visibility, and scintillation will be taken account of in the power tolerance

(b) Background:-------------------------a function of brightness; to be measured in field-of-view

E. SYSTEM AND EQUIPMENT DESIGN PARAMETERS

Simulation runs will be planned to examine the performance of this system for the indicated range of parameter values.

(1) Tolerance in power budget:---------------+5db and -10db
+10db and -20db

(2) Detection range:

(a) Range, R_o, for P_{DET} equal 0.99:-----5,3,2, and 1 miles

(b) Decoding logic:---------------------P_{DET}^2; P_{DET}^3 for IR

(3) False alarm interval:--------------------infinite, 2 minutes

(4) Sampling intervals and processing delays:

(a) Sampling intervals:-----------------0.3, 1.0, and 3.0 seconds

(b) Processing delay:-------------------none

(5) Multiple target discrimination:----------one per bearing resolution element

(6) Angle tracking capability:--------------none, and 3° per second

(7) Operational controls:------------------none, except for threshold level setting with IR

PRELIMINARY SPECIFICATION FOR HYPOTHETICAL PWI V

A. BRIEF DESCRIPTION

PWI V detects all aircraft which intrude into a region of range and altitude proximity. Range-gating depends upon a statistical probability of detection and altitude data. This system measures and indicates the relative bearing of intruder aircraft, and resolves multiple aircraft not in the same bearing resolution element. This system is identical to PWI IV except that the indication of above/below/same altitude is replaced by precise altitude filtering.

B. PRIMARY CLASSIFICATION ACCORDING TO DERIVED DATA

(1) Range:
  (a) Not measured
  (b) P_{DET} versus R
(2) Range-rate:------------------------------none
(3) Bearing:
  (a) Sectors:---------------------------90°, 30°, 10°
  (b) Resolution and accuracy:-----------5° and 2°
(4) Bearing-rate:--------------------------none
(5) Altitude difference:-------------------± 1000 feet
  ± 500 feet
(6) Elevation angle:-----------------------none

C. SPATIAL COVERAGE

(1) Aircraft number 1:
  (a) Bearing:---------------------------360° and 180°
  (b) Elevation:-------------------------$\pm 30^{\circ}$ and $\pm 10^{\circ}$
(2) Aircraft number 2:
  (a) Bearing:---------------------------360°
  (b) Elevation:-------------------------$\pm 30^{\circ}$

D. OPERATING WAVELENGTH

In the simulation experiments we will examine the performance of this system at all three wavelengths of interest.

(1) Microwave:

(a) Propagation:------------------------$1/R^2$

(b) Background:-------------------------NA

(2) Millimeter (55GHz):

(a) Propagation:------------------------$1/R^2 + \alpha$, where α is a function of altitude and precipitation

(b) Background:-------------------------NA

(3) IR:

(a) Propagation:------------------------$1/R^2 + \alpha +$ scintillation; α is a function of visibility and scintillation will be taken account of in the power tolerance

(b) Background:-------------------------a function of brightness; to be restricted to sun saturation

E. SYSTEM AND EQUIPMENT DESIGN PARAMETERS

Simulation runs will be planned to examine the performance of this system for the indicated range of parameter values.

(1) Tolerance in power budget:--------------+ 5db and -10db
+10db and -20db

(2) Detection range:

(a) Range, R_o, for P_{DET} equal 0.99:----5, 3, 2, and 1 miles

(b) Decoding logic:---------------------P^2_{DET}; P^3_{DET} for IR

(3) False alarm interval:-------------------infinite, 2 minutes

(4) Sampling intervals and processing delays:

(a) Sampling intervals:-----------------0.3, 1.0, and 3.0 seconds

(b) Processing delay:-------------------none

(5) Multiple target discrimination:----------one per bearing resolution element

(6) Angle tracking capability:--------------none, and 3^o/second

(7) Operating controls:---------------------Sensitivity control for IR

PRELIMINARY SPECIFICATION FOR HYPOTHETICAL PWI VI

A. BRIEF DESCRIPTION

PWI VI detects all aircraft which intrude into a region of range and altitude proximity. Precise range-gating, based upon an observation or propagation time, and precise altitude filtering, based upon an exchange of barometric altitude data, are provided. This system measures and indicates the relative bearing of intruder aircraft, and it resolves multiple targets not in the same bearing resolution element. This system is identical to PWI V except that the range-gating is precise rather than statistical.

B. PRIMARY CLASSIFICATION ACCORDING TO DERIVED DATA

(1) Range:

(a) Precise range-gate

(b) $R \leq R_o$; $P_{DET} = 1$ and $R > R_o$; $P_{DET} = 0$

(2) Range-rate:------------------------------none

(3) Bearing:

(a) Sectors:----------------------------30^o and 10^o

(b) Resolution and Accuracy:------------5^o and 2^o

(4) Bearing-rate:----------------------------none

(5) Altitude difference:---------------------± 1000 feet
$\pm$ 500 feet
$\pm$ 250 feet

(6) Elevation angle:-------------------------none

C. SPATIAL COVERAGE

(1) Aircraft number 1:

(a) Bearing:---------------------------360^o and 180^o

(b) Elevation:-------------------------$\pm 30^o$ and $\pm 10^o$

(2) Aircraft number 2:

(a) Bearing:----------------------------360^o

(b) Elevation:--------------------------$\pm 30^o$

D. OPERATING WAVELENGTH

In the simulation experiments we will examine the performance of this system at microwave and millimeter wavelengths. The specification of precise range-gating makes IR approaches unlikely except for lasers which are considered as a special-case.

Propagation losses will not influence the results of simulation runs with the PWI and, accordingly, they will not be simulated. In contrast, although background effects will not be simulated, the saturation effect of the sun would influence the results at IR.

E. SYSTEM AND EQUIPMENT DESIGN PARAMETERS

Simulation experiments will be planned to examine the performance of this system for the indicated range of parameter values.

(1) Tolerance in power budget:---------------+ 5db and -10db
+10db and -20db

(2) Detection range:-------------------------selected by operator

(3) False alarm interval:-------------------infinite, 2 minutes

(4) Sampling intervals and processing delays:

(a) Sampling intervals:-----------------0.3, 1.0, and 3.0 seconds

(b) Processing delay:-------------------none

(5) Multiple target discrimination:-----------one per bearing resolution element

(6) Angle tracking capability:---------------none, and 3^{o}/second

(7) Operating controls:-----------------------Range Selector: 1, 2, 3, 4, and 5 miles

PRELIMINARY SPECIFICATION FOR HYPOTHETICAL PWI VII

A. BRIEF DESCRIPTION

PWI VII detects all aircraft which intrude into a region of range proximity. This system measures and indicates the relative bearing and relative elevation and it resolves multiple targets which are not within a common bearing/elevation resolution element. The range and relative bearing measurements are sufficiently precise to permit derivation of range and bearing rates which can be used to reduce the number of alarms on non-threatening targets.

B. PRIMARY CLASSIFICATION ACCORDING TO DERIVED DATA

(1) Range:

(a) Precise range-gate

(b) $R \leq R_o$: $P_{DET} = 1$ and $R > R_o$: $P_{DET} = 0$

(2) Range-rate: -------------------------- 20 feet per second
200 feet per second

(3) Bearing:

(a) Sectors -------------------------- 30^o and 10^o

(b) Resolution and Accuracy ---------- 5^o and 2^o

(4) Bearing-rate ------------------------- 1^o and 3^o per sec.

(5) Altitude difference ------------------ none

(6) Elevation angle ---------------------- above, below, or same altitude (referred to air-frame)

C. SPATIAL COVERAGE

(1) Aircraft number 1:

(a) Bearing-------------------------- 360^o and 180^o

(b) Elevation ----------------------- $\pm 30^o$ and $\pm 10^o$

(2) Aircraft number 2:

(a) Bearing -------------------------- 360^o

(b) Elevation ----------------------- $\pm 30^o$

D. OPERATING WAVELENGTH

In the simulation experiments we will examine the per-

formance of this system in the IR spectral region.

(a) Propagation ----------------------	$1/R^2 + \alpha$ + scintillation; α is a function of visibility, and scintillation will be taken account of in the power tolerance
(b) Background ----------------------	due to direct sun and sun illuminated clouds

E. <u>SYSTEM AND EQUIPMENT DESIGN PARAMETERS</u>

Simulation runs will be planned to examine the performance of this system for the indicated range of parameter values.

(1) Tolerance in power budget: -------	+ 5 db and -10 db +10 db and -20 db
(2) Detection range: -----------------	selected by operator: 1, 2 or 3 miles
(3) False alarm interval: ------------	infinite, one hour
(4) Sampling interval ----------------	3 seconds
Processing delay ------------------	none
(5) Multiple target discrimination:---	one per bearing resolution element
(6) Angle tracking capability:--------	1° and 3° per second
(7) Operating controls: --------------	range selector: 1, 2 and 3 miles

PRELIMINARY SPECIFICATION FOR HYPOTHETICAL PWI VIII

A. BRIEF DESCRIPTION

PWI VIII detects all aircraft which intrude into a region of range and altitude proximity. Precise range-gating, based upon an observation of propagation time, and precise altitude filtering, based upon an exchange of barometric altitude data, are provided. This system measures the range-rate. It does not measure relative bearing. Signals from multiple targets are assumed to be time ordered and resolvable.

B. PRIMARY CLASSIFICATION ACCORDING TO DERIVED DATA

(1) Range:

(a) Precise range-gate

(b) R R_o: $P_{DET} = 1$ and R R_o: $P_{DET} = 0$

(2) Range-rate: ------------------------ 20 feet per sec.
200 feet per sec.

(3) Bearing: --------------------------- none

(4) Bearing-rate ----------------------- none

(5) Altitude difference: --------------- ± 1000 ft.
± 500 ft.
± 250 ft.

(6) Elevation angle: ------------------- none

C. SPATIAL COVERAGE

(1) Aircraft number 1:

(a) Bearing: ------------------------ 360° and 180°

(b) Elevation: ---------------------- ± 90°

(2) Aircraft number 2:

(same as aircraft number 1)

D. OPERATING WAVELENGTH

In the simulation experiments the operation of this system will be examined in the microwave band assuming that atmospheric attenuation is negligible.

E. SYSTEM AND EQUIPMENT DESIGN PARAMETERS

Simulation runs will be planned to examine the performance of this system for the indicated range of parameter values.

(1) Tolerance in power budget: ------- + 5 db and - 10 db
+10 db and - 20 db

(2) Detection range: ---------------- selected by operator
1, 2 or 3 miles

(3) False alarm interval: ---------- infinite, one hour

(4) Sampling interval: ------------- 3 seconds
Processing delay: -------------- none

(5) Multiple target discrimination: - any number

(6) Operating controls: ------------- range selector:
1, 2 and 3 miles

Threat Logic and Alarm Rates in PWI and CAS Equipment, Part I.

V. Mangulis

W. Graham

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Table of Contents

Abstract

Alarm and maneuver rates in a model terminal area and the average time in an alarm condition are calculated for an arriving air carrier flight which is protected by CAS or various PWI equipments. It is assumed that all VFR aircraft have at least the minimal equipment required to enable the air carrier CAS or PWI to function, and that the VFR aircraft have random headings. Only the two-dimensional problem is considered, which means that altitude information is assumed to be exchanged between aircraft. Three different hypothetical PWI devices are considered, such that either a) range only, b) range and bearing, or c) range, range rate, and bearing are available to filter the threat and warn the pilot. The alarm and maneuver rates for the PWI eouipment are compared with the alarm rate for the ATA CAS. The average percentage of time that an arriving air carrier is in a "no turn" CAS alarm condition due to proximity of VFR aircraft, and the expected number of maneuvers due to conflicts with VFR aircraft are calculated as a function of terminal traffic density.

Acknowledgement

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List of Illustrations and Tables

Glossary

a	: maximum lateral acceleration of aircraft.
AC	: air carrier
ATA	: Air Transport Association of America
CAS	: collision avoidance system
E	: elliptic integral
GA	: general aviation
K	: total number of maneuvers during a flight of fixed duration in the terminal area.
M	: total number of alarms during a flight of fixed duration in the terminal area.
N_{C1}	: number of alarms per hour for the CAS τ_1-zone threat logic.
N_{C2}	: " " " " " " " CAS τ_2-zone " " .
N_{P3}	: " " " " " " " PWI-3 " " .
N_{P6}	: " " " " " " " PWI-6 " " .
N_{P8}	: " " " " " " " PWI-8 " " .
n	: density of intruders with a particular velocity magnitude, in number of aircraft per square nautical mile.
n_o	: total density of intruders, all speeds.
PWI	: pilot warning instrument.
PWI-3	: PWI which uses range data only.
PWI-6	: PWI which uses range and bearing data.
PWI-8	: PWI which uses range, range rate, and bearing data.
R	: range from protected aircraft to the intruder.
$\dot{R}$	: range rate, dR/dt

R_a : range parameter used in the PWI-8 threat logic.

R_M : " " " " " CAS τ_1-zone threat logic.

R_o : " " " " " CAS τ_2-zone " " .

R_1 : " " " " " PWI-3 threat logic.

R_2 : " " " " " PWI-6 " " .

S : half-width of the CAS τ_2-zone normal to the relative velocity vector.

T : minimum warning time needed in a PWI system.

T_D : average duration of an alarm.

T_W : average warning time.

t_o : total time spent by an AC aircraft in the terminal area.

u : minimum speed of the protected aircraft.

v_r : relative velocity.

$\bar{v}_r$: average relative velocity.

v_o : maximum aircraft speed.

v_1 : AC aircraft velocity.

v_2 : GA aircraft velocity.

θ : relative heading, see Figure 2.

τ_1 : time parameter used in the CAS τ_1-zone threat logic.

τ_2 : " " " " " CAS τ_2-zone " " .

I. INTRODUCTION

The present Contract has as its primary objective the estimation of the potential benefit to be derived by the various users of the air-space through the implementation of PWI systems of various degrees of sophistication. It is anticipated that the effectiveness of any PWI will depend strongly on a) the rate of alarms it generates, b) whether or not the pilot can detect the targets which cause the alarms, and c) whether or not the pilot considers the targets dangerous[(4)*]. It is proposed to measure the effectiveness of various systems by exercising them through simulation with pilots who are busy with work loads appropriate to their mission. This report relates alarm rates to traffic density for a few of the more sophisticated types of PWI systems that are planned to be simulated.[(5)] The most sophisticated of these PWI systems measures the same quantities as the ATA CAS[(1)(2)] so the alarm rates for the CAS are given for comparison. The analysis is similar to that published by Holt[(6)] but has the following differences and extensions: relative bearing of targets is assumed to be available in some PWI systems, a specific distribution of aircraft speed[(3)] and relative heading is assumed over which the encounter rate is integrated, and the longitudinal component of acceleration of both aircraft is assumed to be zero in the maximum closing speed encounter, i.e., neither aircraft can go faster.

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*References appear on p. 31 .

In comparing CAS with PWI certain differences should be borne in mind: 1) the CAS provides virtually perfect protection between equipped aircraft in principle and can function at any closing speed; 2) all PWI depend on the pilot(s) to see and avoid impending collisions and the ability of pilots to avoid is less than perfect and gets worse with increasing closing speed; 3) an alarm in a PWI, ideally, merely calls the attention of the pilot to a target which he would see anyway if he were looking; he maneuvers only when he considers the situation hazardous; but in the CAS every τ_1 alarm is a command to maneuver.

As a practical matter interest in PWI systems persists, even though the potential protection achieved will be less than that offered by CAS, because of the prospect of building systems at lower cost and thereby achieving wider implementation and greater actual protection. Thus CAS will probably be used primarily to back up the protection afforded IFR/IFR flights by the ATC system. PWI devices will probably be used to help pilots separate IFR/VFR and VFR/VFR traffic. Because of the difference in typical closing speeds, traffic densities and acceptable cost, it is anticipated that the PWI installation designed to protect IFR from VFR traffic will differ from that used to protect VFR from other VFR aircraft. The present report considers only sophisticated PWI systems and these only from the point of view of the IFR user. A subsequent report* will

* The last report in this volume.

cover these systems from the point of view of the VFR user, and will consider less sophisticated PWI systems.

The alarm rates in both CAS and PWI depend on the warning time provided. In the case of the ATA CAS which utilizes vertical maneuvers the time required (25 seconds) is well documented.(7) The warning time in practical PWI systems will probably turn out to be a compromise between excessive alarm rates and inadequate detection and maneuvering time, the details depending on the system. The assumption made in this report is that each PWI system provides a minimum warning time of 15 seconds for the worst case of two aircraft at the maximum legal terminal area speed (250kn IAS, 291kn TAS) with each allowed to have a maximum lateral acceleration of one-half g. This choice of warning time leaves little time in the worst case for the pilot to detect the target and determine if a maneuver is necessary, but it still may be a reasonable value to use because the pilot's ability to detect and evaluate threats beyond the ranges corresponding to these speeds and warning time is probably rather limited.

The choice of a fixed minimum warning time under the worst condition for the various PWI systems analyzed in this report leads to different typical warning times for these systems. A more sophisticated PWI system which measures range rate can permit a target to approach closer in general than a system which measures range only since the latter must assume the range

rate is the highest possible. As a result the less sophisticated systems will have a higher average warning time (since in general the range rate is not the maximum possible) which is an advantage which should be balanced against the lower alarm rate of the more sophisticated system.

In this report the protected aircraft is assumed to be located in a random distribution of other aircraft with a uniform density and random headings in the horizontal plane. For simplicity we consider two dimensional encounters only; in effect this means that altitude data are exchanged or obtained by some other means, and the system considers threats in some co-altitude band only. In this report the effective width of this band is assumed to be ± 800 feet, biased as necessary for high rates of climb or descent.

The threat logics for the various systems are described in Section II, and also in Appendices A and B. The traffic model--air carrier and general aviation velocity distributions in a typical terminal area[3]--is detailed in Section III. The alarm and maneuver rates and the expected time in an alarm condition for this traffic model are obtained in Section III; the mathematical evaluation of alarm rates is described in Appendices A and B. Conclusions are presented in Section IV.

II. THE THREAT LOGIC

The CAS and PWI equipment measure some properties of the intruder (such as range, range rate, or bearing) as specified below, and on the basis of this information classify the intruder as a hazard or not a hazard. The classification process is performed by the threat evaluation logic of the system, as described below. The boundaries of the alarm regions are shown in Figure 1.

A. Collision Avoidance System (CAS).

The threat logic is the one proposed by ATA[1] with parameters as modified by the McDonnell Douglas Corporation[2]. The system measures the range R and the range rate $\dot{R}$. The threats are of two types: a) those which cross the τ_2-zone boundary; the threat logic output then commands the pilot to roll out, if in a turn, and to prepare to climb or dive; b) those which cross the τ_1-zone boundary; the threat logic output then commands the pilot to climb or dive.

The τ_2-zone boundary is given by the following relationship between the range R and the range rate $\dot{R} = dR/dt$:

$$R = R_o - \dot{R}\tau_2, \qquad (1)$$

where $R_o = 1.8$ n. mi. and $\tau_2 = 40$ seconds.

The τ_1-zone boundary is given by either

$$R = -\dot{R}\tau_1 \qquad (2)$$

or

$$R = R_M, \qquad (3)$$

whichever occurs first. $R_M = 0.5$ n. mi. and $\tau_1 = 25$ seconds. For negative values of $\dot{R}$ (closing intruder) the τ_1-zone is contained within the τ_2-zone. The boundaries of the two zones are shown in Figure 1 for the following case: the velocity of the protected aircraft is 176 knots, the velocity of the intruder is 104 knots, and the relative heading is 98° which corresponds to the average relative velocity magnitude of 192 knots.

B. Pilot Warning Instruments (PWI).

a. Range measurement alone (PWI-3).

The intruder is classified as a threat whenever he crosses a circle of radius 14,740 feet, centered on the protected aircraft, see Figure 1. The alarm range to the intruder is the same in all directions since relative bearing is not measured.

As described in Appendix B, the range is chosen in such a way that in the worst case (both aircraft on the same straight line path and heading directly towards each other) 15 seconds remain to a potential collision after the pilot is alerted if both the protected aircraft and the intruder travel with the maximum speed: 291 knots true air speed. For speeds less than maximum or for other headings than the worst case the time to collision (if any) will exceed 15 seconds.

b. Range and bearing measurement (PWI-6).

The intruder is classified as a threat whenever he crosses a circle of radius 10,590 feet, with center 4950 feet ahead of the protected aircraft, see Figure 1. Since the protected air-

craft is not at the center of the circle, the range to the circular boundary of the alarm region changes with the bearing. Consequently, both the range and the bearing have to be obtained to use this threat logic.

As described in Appendix B, the alarm circle is chosen in such a way that in all cases (i.e., for all headings) at least 15 seconds remain to a potential collision after the pilot is alerted if both the protected aircraft and the intruder travel with maximum speed. PWI-3 overprotects* even at the maximum speed for headings other than the worst case. PWI-6 provides 15 seconds of warning at the maximum speed (except for a minor overprotection at some angles because for simplicity we have replaced a rectangle with rounded corners by a circle, see Appendix B), but for speeds less than maximum the time to collision (if any) will exceed 15 seconds.

c. Range, range rate, and bearing measurement (PWI-8).

The intruder is classified as a threat whenever he enters the region with the boundary

$$R = R_a - \dot{R}T , \qquad (4)$$

where $R_a = 3600$ feet and $T = 15$ seconds. Note the similarity of this equation to Eq.(1).

The reasoning behind Eq.(4) is detailed in Appendix B and can be summarized as follows: if both the protected aircraft and

---

* By "overprotection" we mean that the warning time exceeds 15 seconds.

the intruder are on linear flights, then a collision will occur in $T \doteq 15$ seconds if $\dot{R}$ is negative (closing intruder) and $R \doteq -\dot{R}T$. However, if both aircraft can accelerate during this time, then we must add a range $R_a = aT^2$ to our protective boundary (a is the acceleration) which leads to Eq.(4). We obtain $R_a \doteq 3600$ feet for $T = 15$ seconds and $a \doteq 16$ feet/second^2.

The bearing information does not appear explicitly in Eq.(4), and it is not used to classify an intruder as a threat or not a threat. The bearing information is provided to the pilot to help him see and avoid the intruder. Without the bearing information T would presumably have to exceed 15 seconds to give the pilot additional time to locate the intruder.*

The boundary of the alarm region is shown in Figure 1 for the same velocities as CAS. Note that the systems depending on range rate (CAS, PWI-8) have alarm regions which are symmetric about the relative velocity vector v_r, while the system depending on bearing (PWI-6) has an alarm region symmetric about the velocity vector v_1 of the protected aircraft.

If one compares the alarm regions of CAS and PWI-8 (both measure range rate), one should note the following: 1) if acceleration cannot exceed 16 ft/sec^2, then PWI-8 gives a warning at least $T \doteq 15$ seconds prior to a collision, but the τ_2-zone alarm gives less than $\tau_2 \doteq 40$ seconds warning time, even though Eqs. (1) and (4) are similar; the reason for this discrepancy is that R_0 is less than $a\tau_2^2$ (see Appendix A for some typical times to collision); 2) turns are not permitted inside

*If the aircraft accelerate, there is no overprotection; if the aircraft do not accelerate, some warning times will exceed 15 seconds.

the CAS τ_2-zone; therefore at the τ_1-zone boundary one does not need protection against lateral acceleration in turns; 3) it is assumed that a pilot performs an escape maneuver in less time if alerted by a PWI and is free to choose the most appropriate action than if commanded by CAS to perform a climb or dive; an experimental check of this assumption would be valuable.

III. ALARM RATES

We will evaluate the alarm rates for the CAS and the three PWI systems described in the previous section. We will use the velocity distributions of air carrier (AC) and general aviation (GA) aircraft in a typical terminal area[3]. We divide the velocity distribution of each type (AC or GA) into three groups of equal probability, and select the median velocity of each group to represent that group, i.e., the speed mix is assumed to be such that 33% of the aircraft of each type (AC or GA) have one of the velocities shown in Table I. Table II shows the average relative velocity magnitudes between the groups of AC and GA aircraft, if we assume random headings of GA aircraft.

We denote by N_{C1}, N_{C2}, N_{P3}, N_{P6}, and N_{P8} the alarm rates (in number of alarms per hour) for the CAS τ_1-zone, τ_2-zone, the PWI-3, PWI-6, and PWI-8 threat logics, respectively. The details of the alarm rate calculations are presented in Appendices A and B.

Let n be the density (number of aircraft per square nautical mile) of each group of GA aircraft; since each group constitutes one third of the total GA aircraft, the total density n_o of GA aircraft is $n_o = 3n$. Table III shows the alarm rates divided by n (i.e., to obtain the true alarm rates, one has to multiply the numbers presented in Table III by the density n) for encounters between a protected AC aircraft (with one of the three speeds)

Table I. Typical Velocities of AC and GA Aircraft in a Terminal Area.

j	AC $v_1(j)$ knots	i	GA $v_2(i)$ knots
1	141	1	86
2	176	2	104
3	242	3	143

Table II. Average Relative Velocities Between AC and GA Aircraft, Knots.

		GA		
	j \ i	1	2	3
AC	1	154	161	181
	2	186	192	207
	3	250	253	264

Table III.

Alarm Rates

	GA i / AC j	1	2	3
N_{C1}/n *	1	201	224	287
	2	274	299	365
	3	458	482	553
N_{C2}/n	1	754	809	958
	2	963	1015	1150
	3	1433	1482	1619
N_{P3}/n	1	749	781	877
	2	904	930	1002
	3	1211	1228	1278
N_{P6}/n	1	538	561	630
	2	650	668	720
	3	870	883	919
N_{P8}/n	1	260	280	334
	2	334	354	406
	3	503	521	571

* Dimensions are: (alarms/hour)/(aircraft/(n. mi.)^2)

and each of the GA aircraft groups. The entries in Table III can also be used to obtain alarm rates for a protected GA aircraft in encounters with AC aircraft; the alarm rate for the average GA aircraft in the random distribution is given by the entry in Table III multiplied by the density of the AC aircraft group.

If we assume that the AC aircraft spends a time t_o in the terminal area, and one third of the time is spent at each of the three AC speeds in Table I, then for a particular threat logic the total number of alarms M during the time t_o, with all groups of GA aircraft, is given by the sum of the nine entries in Table III (for the particular system), multiplied by n ($n = n_o/3$) and by $t_o/3$. Table IVa shows $M/n_o t_o$ and also M for the near future, when $t_o = 800$ seconds and $n_o = 0.0270$ per $(n.mi.)^2$, numbers estimated to be appropriate for the busiest terminals for the next ten to twenty years[4]. This value of n_o represents 480 aircraft in a terminal area with a radius of 30 n.mi. and height 10,000 feet, if the altitude discrimination of the CAS and PWI equipment is such that only intruders in a layer 1600 feet thick are considered to be hazards.

Table IVa also shows M for current operations in a busy terminal, when there are 113 GA aircraft in the terminal area, $n_o = 0.00636$ per $(n.mi.)^2$.

Table IVa also gives the average duration of alarms T_D for the typical case shown in Figure 1. T_D is the average time the intruder spends within the alarm region, if on a linear unaccel-

Table IV. AC/GA Encounters

in a Terminal Area.

a. Number and Duration of Alarms, and Warning Times.

Threat Logic	$M/n_o t_o$, $(n.mi.)^2/hr.$	Future* M	Current** M	Alarm duration, T_D, secs.	Warning time, T_W, secs.
CAS τ_1-zone	349	2.09	0.49	22	25
CAS τ_2-zone	1131	6.78	1.60	61	73
PWI-3	996	5.97	1.40	72	44
PWI-6	715	4.29	1.01	51	42
PWI-8	396	2.37	0.56	21	26

Dimensions of M: alarms/800 seconds.

b. Number of Maneuvers.

Threat Logic	$K/n_o t_o$, $(n.mi.)^2/hr.$	Future* K	Current** K
CAS τ_1-zone	349	2.09	0.49
CAS τ_2-zone	170	1.02	0.24
All PWI***	135	0.81	0.19

Dimensions of K: maneuvers/800 seconds.

*n_o = 0.0270 GA aircraft per $(n.mi.)^2$.

**n_o = 0.00636 GA aircraft per $(n.mi.)^2$.

t_o = 800 secs.

***For maneuvers assumed miss distance = 2000 ft.

erated flight.

If the geometric situation is such that a collision could actually occur, then for aircraft on linear flights the warning time given by the CAS or PWI equipment is obtained by dividing the distance along the relative velocity vector from the protected aircraft to the alarm region boundary by the relative velocity. This distance/velocity ratio has been calculated at each of the nine average relative velocities given in Table II, the ratios have been summed and divided by nine, and this average warning time T_W is presented in Table IVa.

Let us now consider maneuver rates instead of alarm rates. The protected aircraft must always maneuver if the CAS τ_1-zone is penetrated; therefore, for this threat logic the alarm rate equals the maneuver rate. At the τ_2-zone penetration one must maneuver only if in a turn (roll-out); if we assume that the aircraft turn 15% of the time in the terminal area, then for the CAS τ_2-zone threat logic the maneuver rate is 0.15 times the alarm rate. We assume that the aircraft equipped with any of the PWI will maneuver only if the alerted pilot estimates that the intruder will come within 2000 feet of his aircraft. Thus the maneuver rate is proportional to the area swept out by a circle with diameter 4000 feet, traveling with the relative velocity. We denote the total number of maneuvers during the time t_o by K, and we average over the three AC speeds and three GA speeds the same way as for the total number of alarms. Table IVb shows $K/n_o t_o$ and K for the values of n_o discussed above.

IV. CONCLUSIONS.

The number of alarms, the number of maneuvers, and the average duration of the alarms have been estimated for an arriving IFR air carrier aircraft in potential conflict with random VFR traffic in a terminal area on the assumption that one of various cooperative PWI or the ATA CAS were in use. For illustrative purposes a current terminal density of VFR traffic has been assumed as well as a projected density about four times as great (1980-1990). The PWI studied represent the more sophisticated of those proposed in that it is assumed that altitude data is exchanged between aircraft. The simplest of these PWI also measures range to other aircraft, the second in sophistication measures relative bearing as well, and the third calculates range rate. The performance of these PWI is compared with that of the ATA CAS in the same environment.

The results show that there is little reduction possible in PWI alarm rate by virtue of measuring bearing in addition to range on the assumption that the same warning time suffices in both systems; this is however a poor assumption and it is anticipated that simulation will demonstrate that some indication of bearing is necessary to reduce search time and to achieve a satisfactory detection rate. The measurement of range rate in a PWI in addition to range permits a significant reduction in alarm rate (about 45%) but there is also a reduction in the warning time given to the pilot in the typical encounter; the

reduction is of approximately the same magnitude; as a result it is suspected that simulation will show that there is little net gain in the effectiveness of see-and-avoid by virtue of measuring range rate in a PWI and using it to delay or suppress alarms.

The calculation shows that an IFR air carrier would experience an average of 4.3 alarms and 0.8 maneuvers (to avoid misses with co-altitude traffic of less than 2000 feet in horizontal separation) in an arriving flight in a terminal with four times the current density of VFR traffic if a PWI were in use which gave altitude information to the air carrier and permitted the air carrier to measure range and bearing to the VFR traffic. The PWI alarm in the air carrier would be on about 23% of the time under these conditions. These rates may or may not be considered tolerable; a more fundamental quantity would be the expected collision rate given that the pilot is helped by the PWI to detect targets sooner and more dependably. In order to estimate the effectiveness of see-and-avoid when using PWI it will be necessary to measure (in simulation) the improvement in the probability of detection afforded by the PWI device. Stated another way, these PWI cannot be faulted for the magnitude of the alarm rates; they are only alerting the pilot to traffic he should see anyway.

The CAS τ_2 alarm rate in this environment, assuming all the VFR traffic is equipped, of 6.78 per air carrier arrival gives, with an average alarm duration of 61 seconds, an expectation that the roll-out alarm is on about 40% of the time. If the air carrier is unable to follow ATC vectors when this alarm is

on, it would appear that the CAS could not operate effectively in this environment. The expected number of τ_1 alarms (climb or descend commands) of 2.1 per flight would seem to present less of a problem. If it is assumed that the air carrier is turning 15% of the time in the terminal, the expected number of roll-out maneuvers executed would be about one per flight, making about 3.1 maneuvers per flight due to both τ_1 and τ_2 alarms. This is approximately four times the maneuver rate that would be experienced if pilots using see-and-avoid maneuvered only when the lateral miss distance would be less than 2000 feet.

The alarm and maneuver rates for the CAS and all PWI appear tolerable at the current VFR traffic density.

APPENDIX A. CAS THREAT LOGIC AND ALARM RATES.

The CAS threat logic[1],[2] uses range and range rate data to classify an intruder as a threat whenever the intruder crosses the boundary of the τ_1-zone or τ_2-zone, given by Eqs. (1)-(3). For negative range rate $\dot{R}$ (the intruder is approaching the protected aircraft) the τ_1-zone is contained within the τ_2-zone, see Figure 1. When an intruder crosses the τ_2-zone boundary, the threat logic output commands the pilot to roll out, if in a turn, and to prepare to climb or dive. When an intruder crosses the τ_1-zone boundary, the threat logic output commands the pilot to climb or dive.

Since the aircraft may accelerate and turn when outside the τ_2-zone, and since the range R_o in Eq. (1) is less than $a\tau_2^2$ (where a is the maximum permissible acceleration), collisions could occur in less than τ_2 seconds from the time a τ_2-zone alarm is given, if one did not take evasive action. For example, consider two aircraft with equal speeds, $v = 400$ ft./sec., on parallel courses slightly more than a distance $R_o = 1.8$ n.mi. apart so that the τ_2-zone alarm is not given as long as the aircraft maintain their parallel paths, because the range rate is equal to zero. If both aircraft now turn towards each other simultaneously with circular turns, acceleration $a = 16$ ft./sec.^{2}, then collision occurs a time t later, where

$$\cos(at/v) = 1 - R_0 a/2v^2 \qquad (A - 1)$$

if no evasive action is taken. For this case $t \simeq 27.5$ seconds.

Of course, the CAS τ_2-zone threat logic will command both pilots (if both are equipped with CAS) to roll out, if the alarm is given. If it takes 10 seconds to accomplish the roll-out, then instead of being on parallel courses the two aircraft will be on intercepting courses, and a collision may occur 29 seconds after roll-out (39 seconds after the alarm), if no other evasive action is taken. The τ_1-zone logic will prevent this collision by commanding the pilots to climb or dive, and normally there are 25 seconds of warning time to accomplish the maneuver.

However, if the intruder is not equipped with CAS* he may elect to turn towards the protected aircraft at the τ_1-zone boundary. For two 400 ft./sec. aircraft and acceleration 16 ft./sec.^{2} in the worst case there are only 13.5 seconds left between the τ_1-zone alarm and the potential collision. The worst case occurs when the intruder approaches the protected aircraft at a relative heading of $\theta = 163°$ (see Figure 2 for a definition of θ) and turns towards the protected aircraft at the minimum range R_M.

Let us now consider alarm rates; we will examine the rates for the τ_2-zone first. For a random distribution of intruder headings let us study the intruders with relative headings at angles between some θ and $\theta + d\theta$ (see Figure 2 for θ) and a

* He carries only a cooperative element and does not get alarms himself.

constant velocity magnitude v_2. If v_1 is the velocity magnitude of the protected aircraft, then the magnitude of the relative velocity v_r is given by

$$v_r^2 = v_1^2 + v_2^2 + 2v_1 v_2 \cos\theta \; . \qquad (A - 2)$$

To obtain the alarm rate we can assume that the intruders are stationary and the protected aircraft moves with velocity v_r. The τ_2-zone boundary is now given by

$$R = R_o + v_r \tau_2 \cos\beta \; , \qquad (A - 3)$$

where β is the angle between the relative velocity vector and the range vector, see Figure 2. Eq. (A - 3) describes a limaçon of Pascal, shown in Figure 2.

Let 2S be the maximum width of the τ_2-zone boundary in Figure 2 normal to the relative velocity vector. It can be shown that

$$S = (z - R_o)^{1/2}(z + 3R_o)^{3/2}/16 v_r \tau_2 \qquad (A - 4)$$

where

$$z = \left[R_o^2 + 8(v_r \tau_2)^2 \right]^{1/2} . \qquad (A - 5)$$

Then the τ_2-zone boundary, moving with velocity v_r, in a time interval Δt sweeps out an area

$$A = 2Sv_r \; \Delta t. \qquad (A - 6)$$

To obtain the number of alarms received in the time interval Δt,

we must multiply A by the density of intruders. If the total density of intruders with this given magnitude v_2 is n, then the density of intruders with headings between θ and $\theta + d\theta$ is $nd\theta/2\pi$, if headings are random. The total alarm rate N_{C2} is obtained by integrating $And\theta/2\pi$ over all θ, and by dividing the result by Δt, or

$$N_{C2} = (n/\pi) \int_0^{2\pi} d\theta \; v_r S \qquad (A - 7a)$$

$$= (n/8\pi\tau_2) \int_0^{\pi} d\theta (z + 3R_o)^{3/2} (z - R_o)^{1/2}. \qquad (A - 7b)$$

To evaluate the integral in Eq. (A - 7b) numerically we approximate it by the use of Simpson's Rule, applied to values of the integrand at the three points $\theta = 0$, $\pi/2$, and π,

$$N_{C2} \approx (n/48\tau_2) \left[F(z_o) + 4F(z_1) + F(z_2) \right] \qquad (A - 3)$$

where

$$F(z) = (z + 3R_o)^{3/2} (z - R_o)^{1/2} \qquad (A - 9)$$

and z_o, z_1, z_2 is the value of z, given by Eq. (A - 5), at $\theta = 0$, $\pi/2$, π respectively.

To check on the accuracy of the approximation, we have compared it with two special cases of the exact Eq. (A - 7b) when the latter can be integrated. In the first special case let $R_o = 0$; then it turns out that both the exact and the approximate expressions give the same result (i.e., there is no error due to approximation):

$$N_{C2}(R_o = 0) = n\tau_2(v_1^2 + v_2^2). \qquad \text{(A -10)}$$

In the second special case we let $\tau_2 = 0$, then exactly

$$N_{C2}(\tau_2 = 0) = n2R_o\bar{v}_r, \qquad \text{(A -11)}$$

where $\bar{v}_r$ is the average relative velocity,

$$\bar{v}_r = (1/2\pi)\int_0^{2\pi} d\theta v_r$$

$$= (2/\pi)(v_1 + v_2)\, E\left[4v_1v_2/(v_1 + v_2)^2\right], \qquad \text{(A -12)}$$

and E is the complete elliptic integral of the second kind. In this special case we find the following percent differences between the exact and the approximate result: 0.23% for $v_1/v_2 = 1$; 1.50% for $v_1/v_2 = 2$ or 1/2; 0.48% for $v_1/v_2 = 4$ or 1/4.

Since the errors in the approximation were very small, the approximation was used to obtain numerical results.

Now consider τ_1-zone alarms. In Eq.(A -7a) we now must let S equal to either R_M or $v_r\tau_1/2$, whichever is greater; they are equal when $\theta = \theta_M$,

$$\cos\theta_M = \left[(2R_M/\tau_1)^2 - v_1^2 - v_2^2\right]/2v_1v_2. \qquad \text{(A -13)}$$

Thus

$$N_{C1} = (n/\pi)\left\{\int_0^{\theta_M} d\theta\, v_r^2\tau_1 + \int_{\theta_M}^{\pi} d\theta\, v_r 2R_M \right. \qquad \text{(A -14)}$$

which becomes

$$N_{C1} = n\left\{(\tau_1/\pi)\left[(v_1^2 + v_2^2)\theta_M + 2v_1v_2\sin\theta_M\right]\right.$$

$$+ 2R_M\left[\bar{v}_r - (2/\pi)(v_1 + v_2)E(\tfrac{1}{2}\theta_M/\alpha)\right] \qquad \text{(A -15)}$$

where $E(\frac{1}{2}\Phi_M / \alpha)$ is the incomplete elliptic integral of the second kind, and

$$\sin\alpha = 2\sqrt{v_1 v_2}/(v_1 + v_2). \qquad \text{(A -16)}$$

The elliptic integrals are tabulated, see, for example, M. Abramowitz and I. A. Stegun, ed., Handbook of Mathematical Functions (Dover Publications, Inc., New York, 1965).

APPENDIX B. PWI THREAT LOGIC AND ALARM RATES.

PWI-3.

We assume that only range data are available. Consequently, since one has to protect the aircraft against the worst possible case-- a head-on collision at maximum speeds--one must set the alarm range R_1 equal to $2v_oT$, where T is the necessary warning time and v_o is the maximum speed. For $T = 15$ seconds and $v_o =$ 291 knots = 491 feet/second we have $R_1 = 14{,}740$ feet.

PWI-3 corresponds to the hypothetical PWI III for which preliminary specifications were given in an earlier report[5].

To evaluate the alarm rate N_{P3} we must substitute for S in Eq.(A -7a) the constant radius R_1, which then yields

$$N_{P3} = n2R_1\bar{v}_r \, , \qquad (B - 1)$$

where $\bar{v}_r$ is the average relative velocity, given by Eq.(A - 12), and n is the density of intruders.

PWI-3 provides more than 15 seconds warning for all cases except the head-on collision at maximum speeds.

PWI-6

Both range and bearing of the intruder are assumed to be available. This system corresponds to the hypothetical PWI VI described in an earlier report.[5]

To derive the threat logic, consider the possible position

of the protected aircraft T seconds later, where T is the needed warning time. Let v_0 be the maximum speed, u be the minimum speed, and a the maximum lateral acceleration. Then the protected aircraft after a time interval T should be inside the rectangle shown in Figure 3 (a good approximation for the speeds and accelerations which we are considering), with sides $(v_0 - u)T$ and aT^2. We are assuming that the same equipment is used by all protected aircraft, and that the threat logic is not adjusted with the speed of the protected aircraft. Consequently, we do not know precisely where the protected aircraft will be T seconds later, and we must provide protection for all possible positions within the rectangle.

We must protect against the intruder with the maximum speed, thus to reach the rectangle in T seconds the intruder must be within a distance D from the rectangle, where approximately

$$D^2 = (v_0T)^2 + (\tfrac{1}{2}aT^2)^2 . \qquad (B - 2)$$

If we draw a contour of the distance around the rectangle in Figure 3, we obtain the rectangle with rounded corners. For simplicity we replace this rectangle with rounded corners by a circle with center at the center of the rectangle and radius R_2,

$$R_2 = D + \tfrac{1}{2}\sqrt{\left[(v_0 - u)T\right]^2 + (aT^2)^2} , \qquad (B - 3)$$

as indicated in Figure 3. The simpler circle overprotects slightly at some angles; therefore, for example, in Figure 1 the

PWI-6 alarm region boundary extends beyond the PWI-3 alarm region boundary in the forward sector. The boundary in Figure 1 is drawn for $a = 16$ ft./sec.^{2}, $T = 15$ seconds, $v_0 = 291$ knots, $u = 100$ knots. Then $R_2 = 10{,}590$ feet.

For speeds slower than maximum this alarm circle provides a longer warning time. Table V shows the minimum warning time obtained if the intruder and the protected aircraft both have one of the speeds listed in Table I. The times are calculated from Eq.(B - 2) with the intruder speed replacing v_0. The difference between the 42 second warning time shown in Table IVa and the values shown in Table V is the following: T_w in Table IVa is the average warning time in encounters between one AC and one GA aircraft, while the times shown in Table V are the worst case minimum times in encounters between AC or GA pairs of aircraft with the same speed.

To evaluate the alarm rate N_{p6} we substitute R_2 for S in Eq. (A - 7a); thus, analogous to Eq. (B - 1), we obtain

$$N_{p6} = n2R_2\bar{v}_r. \qquad (B - 4)$$

Obviously the ratio of N_{p3} to N_{p6} is equal to the ratio of R_1 to R_2.

PWI-8.

Range, range rate, and bearing of the intruder are assumed to be available. However, only the range R and range rate $\dot{R}$ are

Table V. Minimum Warning Times for PWI-6, Intruder and Protected Aircraft Have the Same Speed.

Speed, knots		Warning time, seconds.
	141	24.6
AC	176	22.0
	242	17.5
	86	28.2
GA	104	27.1
	143	24.4

used in the threat logic. The dependence of the threat logic on the bearing is implicit and not explicit because without providing the bearing information to the pilot one would presumably have to use a longer warning time T to permit the pilot to search for the intruder. The addition of explicit bearing information to the threat logic seems to accomplish little. Data shown by Holt and Marner[6] indicate that the use of bearing information in the threat logic would reduce the alarm rate by less than 5% at the average relative speed of 192 knots.

If both the protected aircraft and the intruder are on constant speed linear courses, then the time to collision is given by $-R/\dot{R}$. Thus we would let $R = -\dot{R}T$ as the boundary of the alarm region if we needed a warning time T and if we did not have to worry about acceleration. However, to protect against possible acceleration we will add an extra range R_a to this alarm region. Acceleration along the flight path is negligible compared to lateral acceleration in a turn; consequently, the worst case is the one mentioned in Appendix A: both aircraft on parallel paths a distance slightly greater than R_a apart, both with the same velocity v, so that $\dot{R} = 0$. If both aircraft turn towards each other simultaneously, then they will collide in T seconds, where

$$\cos(aT/v) = 1 - R_a a/2v^2. \qquad (B - 5)$$

If v is sufficiently large so that $aT/v < 1$, we can approximate $\cos x$ by $1 - x^2/2$, where $x = aT/v$, which leads to

$$R_a = aT^2, \tag{B - 6}$$

which is the distance which we need to get a warning time T if the maximum acceleration is a. For a = 16 ft./sec.^{2} and T = 15 seconds we must have R_a = 3600 feet. The boundary of the alarm region is given by Eq.(4). Thus the threat logic is similar to that of the CAS τ_2-zone, but the values of the parameters are different.

Similarly, the alarm rate N_{p8} is given by an equation of the same form as (A - 7) or (A - 8), if we substitute R_a for R_o and T for τ_2.

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* The first report in this volume.

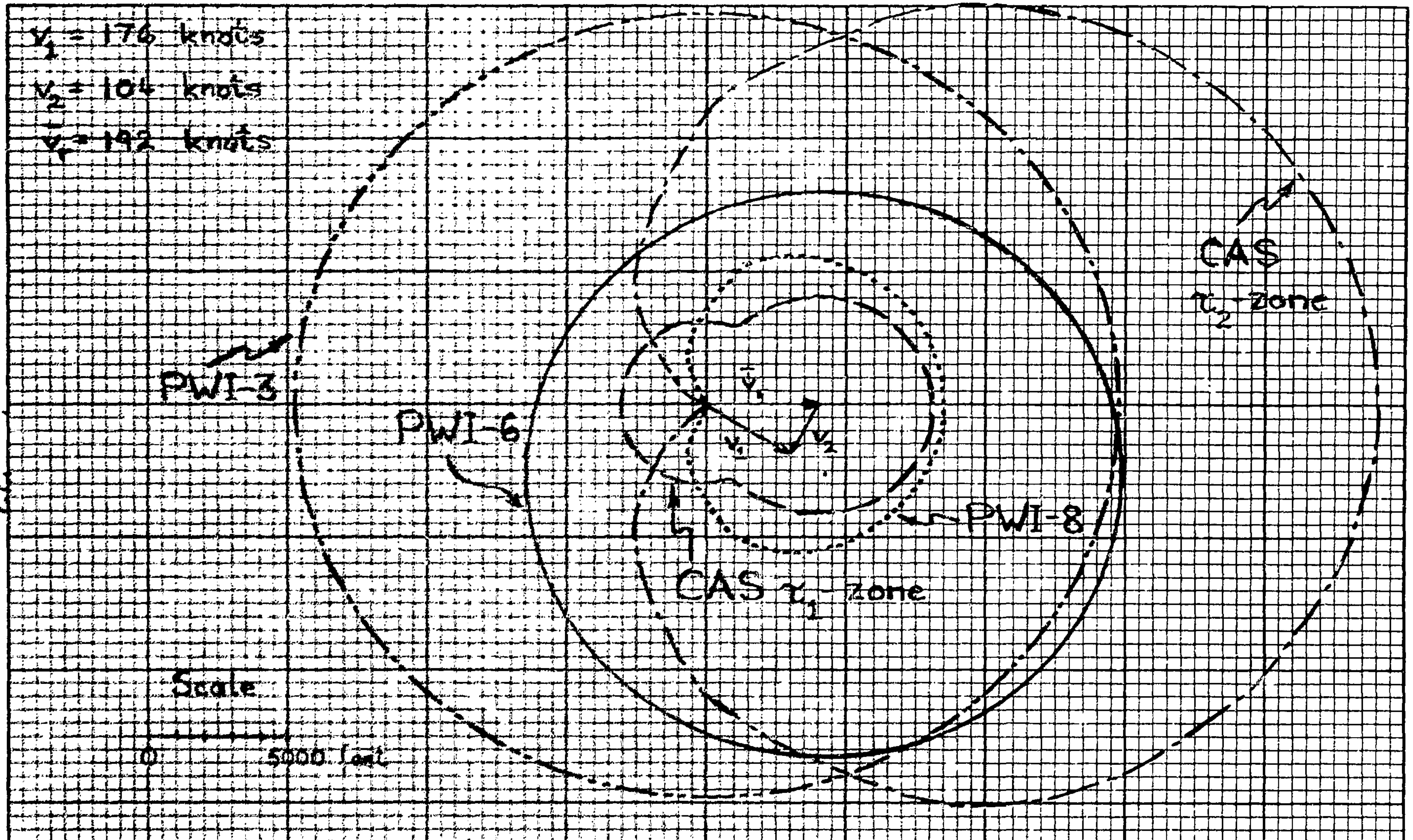

Figure 1. Alarm region boundaries at average relative velocity

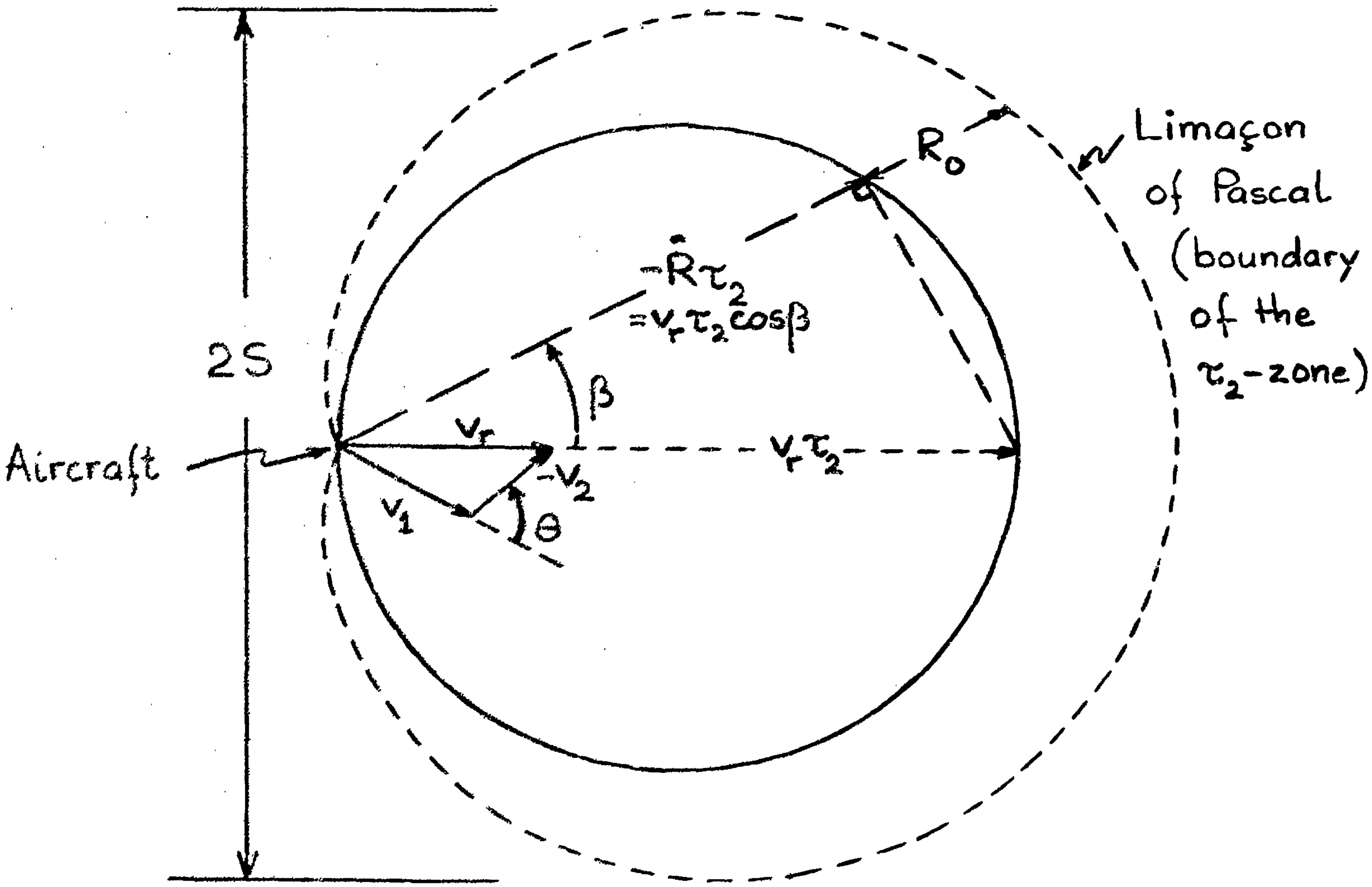

Figure 2. CAS τ_2-zone boundary

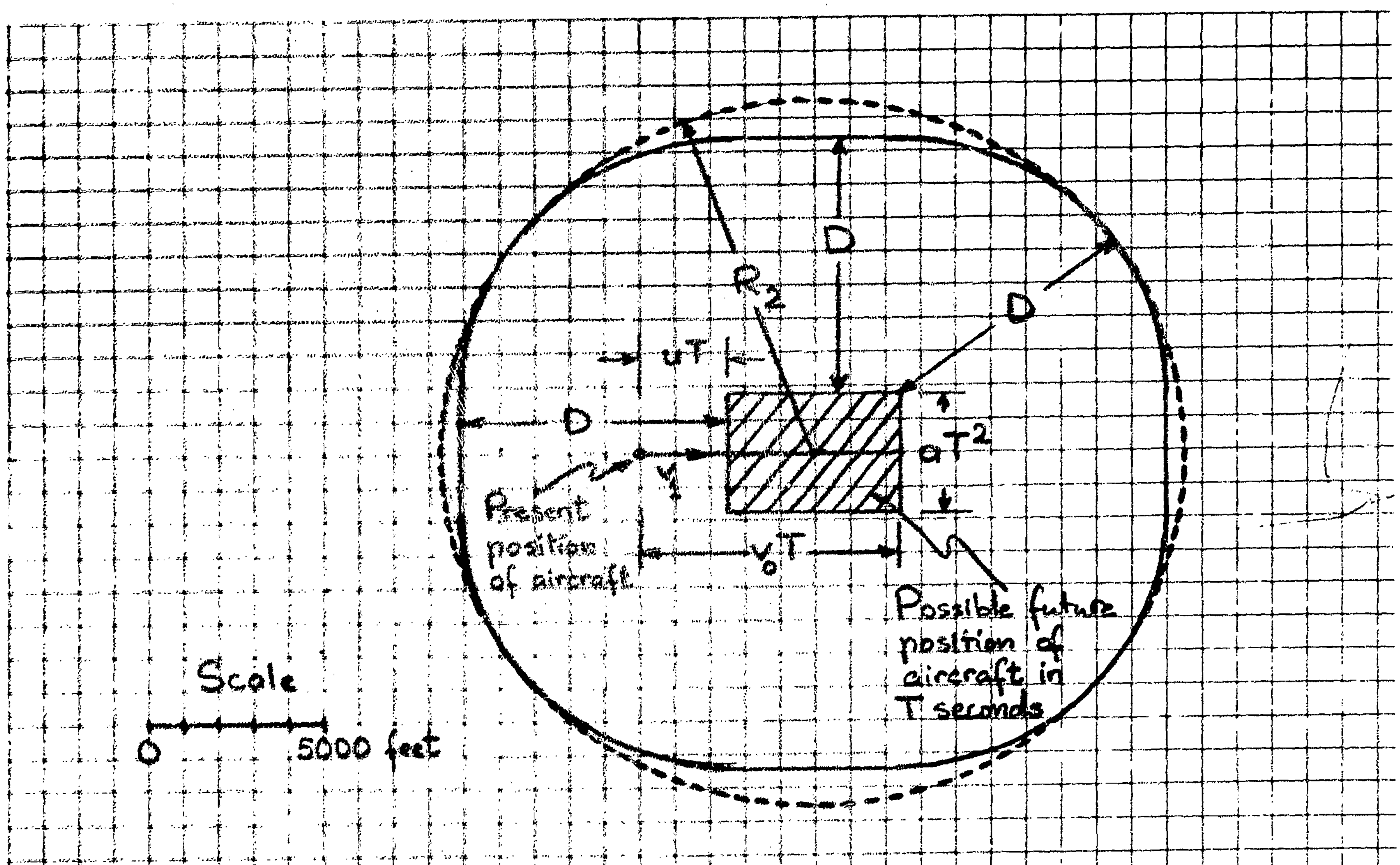

Figure 3. PWI-6 alarm region boundary.

Threat Logic and Alarm Rates in

PWI and CAS Equipment, Part II.

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W. Graham

Report No. CDC-VM-3

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Table of Contents

Abstract

Alarm rates* are calculated for certain cooperative PWI systems of the type in which range is inferred from received signal strength. The role of atmospheric attenuation is investigated in particular; a microwave PWI (operating at a frequency of zero attenuation) and a millimeter wave PWI (operating at a frequency of high attenuation) are compared. Propagation at infra-red falls somewhere between these two depending on atmospheric conditions. To get concrete results the anticipated alarm rates and typical warning times are calculated for air carrier flights encountering random general aviation traffic in terminal environments; these rates can be compared directly with those given in Part I of this report for PWI systems capable of precise range measurement.

* Alarm rate as used in this report is defined as the rate of encounters with other aircraft which cause the generation of alarms.

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List of Illustrations and Tables

Glossary

AC	:	air carrier
ATA	:	Air Transport Association of America
CAS	:	collision avoidance system
F_m	:	a function related to P_n, see Eq.(B-2).
GA	:	general aviation
G_m	:	a function related to P_n, see Eq.(B-3).
IAS	:	indicated aircraft speed.
k	:	$(R_o-R_p)/2v_rT_p$
m	:	$\log_{10}e$
M	:	total number of expected alarms during a flight of 800 seconds in a typical terminal area.
n	:	density of GA intruders with a particular speed, in number of aircraft per square nautical mile.
n_o	:	total density of GA intruders, all speeds.
N	:	alarm rate
p(u)	:	probability density function for signal strength.
$p_1(v_1)$	:	" " " " AC speeds.
$p_2(v_2)$	:	" " " " GA speeds.
p_j	:	probability of not detecting an intruder in the j^{th} range interval, Eq.(A-4).
P_{fa}	:	probability of false alarm.
P_n	:	probability of not obtaining two consecutive successes in n trials, Appendix B.
PWI	:	pilot warning instrument.
PWI-3	:	PWI which uses range data only, Reference 1.

PWI-8 : PWI which uses range, range rate, and bearing data, Reference 1.

$P(w)$: cumulative probability that an intruder will be detected at least once during a flight passing the protected aircraft, if the distance of closest approach is w, and all aircraft are on linear courses.

$P_F(R)$: probability that detection will occur first at range R.

$P_o(R)$: probability of failure to detect the intruder by the time he has reached range R, aircraft on collision course.

$P_1(R)$: $1-P_o(R)$

$\bar{P}_1(R)$: $P_1(R)$ averaged over the signal strength distribution.

Q : Marcum's Q-function, References 2 and 3.

r : projection of range to intruder on the relative velocity vector, see Figure 12.

r_m : maximum value of r, analogous to R_o.

R : range to intruder

R_j : range to the center of the j^{th} range interval, if the entire flight path is divided into range intervals of length $\Delta R = 2v_r T_p$.

R_o : maximum value of range from which one obtains a significant contribution to the cumulative probability of detection.

R_F : mean range of first detection for a constant signal level.

R_p : a design parameter for the PWI; the range at which we require that the cumulative probability of detection should equal 95% for the worst case of head-on approach at maximum speeds.

S : signal power received from any range.

S_o : signal power from range R_p at the peak of the probability density.

S/N : signal-to-noise ratio in general.

S_o/N : " " " " at the peak of the signal strength distribution received from range R_p.

t_o : typical length of time spent in the terminal area by AC aircraft during descent, assumed to be 800 seconds.

TAS : true aircraft speed.

T_m : minimum warning time needed in a PWI system, assumed to be 15 seconds.

T_p : time interval between reception of pulses; the reciprocal of pulse rate.

T_w : mean warning time for a constant signal level and any relative velocity.

$\bar{T}_w$: T_w at the average relative velocity, averaged over the signal strength distribution.

u : $10 \log_{10} S/S_o$, where S is evaluated at R_p.

$U_m(x)$: Tchebyscheff polynomial.

v_r : relative velocity magnitude.

v_1 : AC aircraft speed.

v_2 : GA aircraft speed.

V : R_F/T_m.

w : distance of closest approach during linear flights, aircraft not on a collision course, see Figure 12.

w_o : maximum value of w, analogous to R_o.

Z : normal or Gaussian probability density function.

α : atmospheric attenuation coefficient.

ΔR : range interval $2v_rT_p$.

η : combination of parameters defined by Eq.(D-3).

θ : relative heading.

θ_L : defined by Eq.(D-13).

σ_+ : standard deviation for $u > e$, see Eq.(1).

σ_- : " " " $u < e$, " " .

τ_2 : time parameter used in the CAS τ_2-zone threat logic, see Reference 1.

Φ : probability that the warning time will be less than τ_m = 15 seconds.

$\Phi_{v_r}(V)$: probability that v_r will be less than V.

I. INTRODUCTION

PWI alarm rates, although quantitative, do not fully reflect the relative merits** of various systems because the effective utilization of the information given to the pilot by the PWI will depend not only on the number but the nature of the alarms. Is the target visible? Does it appear to the pilot to be a threat or a potential threat? Does the pilot consider the alarm premature? Etc. Such questions will be answered in the forthcoming simulation program. The alarm rates, however, may be indicative of the relative performance** of PWI systems, and the mathematical tools needed to calculate alarm rates are required anyway to carry out the simulation experiments. For these reasons the present report and its companion (Reference 1.) * have been prepared.

In Reference 1 we investigated the more sophisticated PWI systems (those in which a precise range measurement is made), and we compared the alarm rates and warning times with those of the ATA CAS. In this report we consider less sophisticated PWI systems in which range is not measured directly but the possibility of a threat is inferred from the received signal level which, on the average, falls off with the range between the transmitters and receivers. In these systems an alarm is given when the received signal level exceeds some preset or pilot adjusted threshold value one or more times (altitude filtering may also be used). Systems of this type are attractive because of the potential simplicity of the minimum equipment for light aircraft, but their practicality has been doubted because of anticipated high rate of unnecessary alarms.

The probability of detection is not only a function of the range to the intruder but also of signal strength, and the latter may change due to fluctuations

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* References appear on p. 24.

** "Perfomance of a PWI system" is used in the restricted sense of alarms generated; "merits of various systems" is intended to include dependence on human factors.

in the antenna pattern, multipath effects, changes in power output, etc. As described in the next section, we have assumed some particular probability distributions for the signal strength, and for each distribution we require that the cumulative probability of detection equal 95% at some range R_p for the worst case of head-on approach by two aircraft at the maximum legal terminal area speed (250 knots IAS, 291 knots TAS). We then calculate the expected number of alarms in a typical flight in a terminal area, the average warning time, and the probability that the warning time will be less than 15 seconds; the latter was assumed to be the minimum needed warning time in Reference 1. The calculations are performed for microwaves, for which signal power is assumed to decrease as $1/R^2$, where R is the range, and for millimeter waves, for which in addition to the $1/R^2$ decrease there is a propagation loss due to atmospheric attenuation. As in Reference 1, the protected aircraft is assumed to be located in a random distribution of other aircraft with a uniform density and random headings in the horizontal plane.

The next two sections present the mathematical model in more detail; mathematical derivations are given in appendices. The results are described in Section III, and conclusions are presented in Section IV.

II. Threat Logic

We assume that the PWI equipment receives from the intruder pulses at time intervals T_p, and that the intruder is detected (i.e., an alarm is given to the pilot) if the receiver outputs exceed a threshold for two pulses in succession. For the numerical calculations we chose the threshold in such a way that the probability of false alarm equals 10^{-10} for the double pulse, or 10^{-5} for the single pulse. For a constant received power the probability that the receiver output will exceed the threshold on two successive pulses is given by the square of Marcum's Q-functions[2] which are tabulated.[3]

In practice, even if the intruder is at a fixed range, the received signal strength S will vary mainly due to fluctuations in the antenna pattern, but also due to multipath effects, changes in power output, etc. For simplicity we assume that the signal received from a constant range R_p can be described by the following asymmetrical log-normal probability density function (p(u)du is the probability that u has a value between u and u + du) :

$$p(u) = \begin{cases} 2Z(u/\sigma_+)/(\sigma_+ + \sigma_-) , & u > 0 ; \\ 2Z(u/\sigma_-)/(\sigma_+ + \sigma_-) , & u < 0 ; \end{cases} \qquad (1)$$

where

$$Z(x) = (1/\sqrt{2\pi})e^{-x^2/2} \qquad (2)$$

and

$$u = 10 \log_{10} S/S_o , \text{ at } R_p . \qquad (3)$$

S_o is the signal strength at the peak of the probability density function; however, due to the asymmetry it is not the mean signal strength (note that we are using two different standard deviations σ_+ and σ_- for signals above and below S_o). Figure 1 shows p(u) vs. u for a) $\sigma_+ = 2.5$ db, $\sigma_- = 5$ db;

b) $\sigma_+ = 5$ db, $\sigma_- = 10$ db.

We fix the value of S_o (or rather the signal-to-noise ratio S_o/N) as outlined below and as described in more detail in Appendix A. For the worst case of head-on approach by two aircraft at the maximum legal terminal area speed (250 knots IAS, 291 knots TAS) we require that the cumulative probability of detection should equal 95% by the time the intruder has reached the range R_p, if we average over the signal density in Figure 1. In the subsequent numerical calculations we chose two values of R_p: a) 1 n.mi. = 6080 feet; b) 14,740 feet; at the latter range 15 seconds remain to collision for the worst case.

The variation of the signal strength with range R is assumed to be given by

$$e^{-\alpha R}/R^2$$

where α is the attenuation coefficient. Numerical calculations were performed for: a) microwaves, $\alpha = 0$; b) millimeter waves, $\alpha = 6.8$ db/n.mi., a typical value for oxygen absorption at 55 GHz.[4] The attenuation coefficients for infrared radiation fall inbetween those for microwaves and millimeter waves[4]; consequently, numerical calculations for infrared were not performed.

Figures 2-9 show the probability of detection (the square of the Q-function) vs. range for the two signal strength distributions: 1) $\sigma_+ = 2.5$ db and $\sigma_- = 5$ db, 2) $\sigma_+ = 5$ db and $\sigma_- = 10$ db; for the two design ranges: 1) $R_p = 6080$ feet, 2) $R_p = 14{,}740$ feet; and for the two types of radiation: 1) microwaves, 2) millimeter waves. Figures 2-9 show that the microwave PWI will detect many more intruders at greater ranges than the millimeter wave PWI; this is a disadvantage because detection at great ranges needlessly alarms the pilot. For example, the 50% point on the $S_o + 2\sigma_+$ curve in Figure 5 for microwaves is at 340,000 feet, while in Figure 9 for millimeter waves it is at 34,000 feet, a ratio of 10:1. In practice one needs protection out to about

15,000 feet, see Reference 1, Figure 1; however, if instead of using a precise range gate one relies on the natural decrease of probability of detection with range, then, to insure a high probability of detection at about 15,000 feet, one has to suffer frequent detection beyond this range, but for microwaves due to the lack of atmospheric attenuation this detection extends much further than for millimeter waves.

Note that the indicated values of S_o/N are for the signal received at the protected aircraft from a target at the range R_p. For the same value of S_o/N the intruder must transmit more power if millimeter waves are used, since the propagation losses are much higher than for microwaves. The noise figures of receivers are also somewhat higher at millimeter wave frequencies which increases the power requirement further.

III. Alarm Rates and Warning Times

Table I presents some results which are analogous to those shown in Table IV of Reference 1 as well as some other calculations which did not appear in Reference 1 but are needed here because of the statistical nature of detection. The table shows the results for air carrier (AC) and general aviation (GA) encounters in a typical terminal area for the same two signal strength distributions, two design ranges, and two types of radiation (microwaves and millimeter waves) as those illustrated in Figures 2-9.

We define the average warning time $\bar{T}_w$ as the mean range of first detection averaged over the signal strength distribution and divided by the average relative velocity as explained in Appendix D. We used the median relative velocity of Table II in Reference 1, 192 knots, as the average relative velocity, and the pulse rate $1/T_p = 2$/sec.

We also calculate the probability Φ that the warning time (the range of first detection divided by the relative velocity) may be less than 15 seconds, as described in Appendix D. The cumulative distribution function for the magnitude of the relative velocity is obtained from data presented in Reference 5 for a typical terminal area and is shown in Figure 10.

The calculation of alarm rates is explained in Appendix C. The quantity N shown in Table I is the average number of alarms generated at an AC aircraft by GA intruders during a flight lasting 800 seconds in a terminal "area" with radius 30 n.mi. and height 10,000 feet. For current operations we assume 113 GA aircraft in this terminal area, and for future operations 480 GA aircraft. The GA aircraft are assumed to be uniformly distributed between altitudes 1000 feet and 10,000 feet. The PWI either has no altitude discrimination at all, or it considers threats only in a co-altitude band of ± 800 feet; N for both possibilities are shown in Table I. In Reference 1 we assumed an altitude

Table I. AC/GA Encounters in a Terminal Area

Design range R_p, feet	Standard deviations of power, db σ_+	σ_-	S_o/N db	Average warning time $\bar{T}_w$, secs.	Probability that $T_w \leq 15$s. Φ, in %	Altitude discrimination, feet	Future M *	Current M *
a. Microwaves								
6080	2.5	5	20.6	61	2.7	1600	16	3.8
						none	90	21
	5	10	29.4	192	3.1	1600	151	36
						none	850	200
14,740	2.5	5	19.7	143	0.07	1600	38	9.0
						none	216	51
	5	10	28.5	451	0.57	1600	370	88
						none	2100	500
b. Millimeter waves								
6080	2.5	5	21.4	31	5.4	1600	5.2	1.2
						none	29	6.9
	5	10	30.2	43	3.6	1600	9.0	2.1
						none	50	12
14,740	2.5	5	21.0	61	0.001	1600	9.4	2.2
						none	53	12
	5	10	29.7	75	0.04	1600	14	3.2
						none	76	18

* Dimensions of M: alarms/800 seconds.

discrimination of ± 800 feet.

The simpler the PWI equipment and the threat logic, the more unnecessary alarms will be produced. Thus the number of alarms for the most sophisticated PWI, the PWI-8 in Reference 1, can serve as a standard of comparison. Current M for PWI-8 equals 0.6. The advantage of operating at a frequency of high atmospheric attenuation is shown by comparing millimeter wave systems with microwave systems since the latter produce more unnecessary alarms. Comparison with data presented in Table IV of Reference 1 shows that the millimeter wave system with R_p = 6080 feet, σ_+ = 2.5 db, σ_- = 5 db, and altitude discrimination 1600 feet, generates even fewer alarms than the CAS τ_2-zone or PWI-3 in Reference 1; however, this millimeter wave system has a probability of 5.4% that the warning time will be less than 15 seconds; the same probability is zero (excluding malfunctions) for the more sophisticated systems in Reference 1.

The millimeter wave system with R_p = 14,740 feet, σ_+ = 2.5 db, σ_- = 5 db, has a low probability (0.001%) that the warning time will be less than 15 seconds. For altitude discrimination of 1600 feet it generates four times as many alarms as the most sophisticated PWI-8, but less than twice as many as PWI-3 or the CAS τ_2-zone. An intermediate choice of R_p is clearly indicated.

Note that some entries for M in Table 1 exceed the assumed number of aircraft in the terminal area. This occurs because detection for those systems extends beyond the assumed 30 n.mi. radius of the terminal area, and alarms are generated by aircraft beyond this radius (for the purposes of the calculation we really assume a continuous distribution of GA aircraft over the whole area covered by the PWI equipment).

IV. Conclusions

This analysis shows that it is possible in principle to achieve an alarm rate in a simple PWI system, in which the possibility of a threat is inferred from signal strength, which is only slightly higher than that of a system which measures range precisely if advantage is taken of a high atmospheric attenuation rate to accelerate the fall-off of signal level with range. This can be done at the expense of an occasional very high speed encounter in which there will be less than 15 seconds warning time and at the expense of an increased signal level. The analysis also shows that systems which do not exploit a high atmospheric attenuation rate will have very high relative rates of alarm if allowances are made in the power budget for adverse antenna patterns, multipath propagation, low transmitter power, high receiver noise levels, etc.

The probabilistic analysis given is suitable for application to the problem of appropriate activation of displays in simulation of systems of this type.

In the absence of adequate flight test data it was necessary to approximate the probability density function of received signal-to-noise ratio by assumed distributions; two such distributions were assumed which it is thought will cover the range to be found in practice. It is important, however, to get actual flight measurements on which to base new calculations or simply confirm the validity of the numerical work reported here.

Appendix A. Cumulative Probability of Detection, Intruder on a Collision Course.

We assume that the signal strength variations are mainly due to the fluctuations in the antenna pattern as a function of bearing. On a collision course the bearing remains constant, therefore, the signal strength is assumed to remain constant during a particular encounter. We will obtain the cumulative probability of detection for a constant signal strength as the intruder approaches from a very large distance R_o to a distance R_p. However, since the signal strength may change from encounter to encounter or from one antenna to another, we will average this cumulative probability of detection over the signal strength distribution shown in Figure 1, and we will use this average to determine the signal strength S_o by requiring that this average cumulative probability of detection equal 95% at R_p for the case of head-on approach by two aircraft at the maximum legal terminal area speed (250 knots IAS, 291 knots TAS; consequently, range rate is 582 knots).

For a single pulse, the probability that the signal will exceed a threshold (determined by the probability of false alarm P_{fa}) is given by[2]

$$p_{thr} \doteq Q\left[\sqrt{2S/N},\ \sqrt{2\cdot \ln(1/P_{fa})}\right] \qquad \text{(A-1)}$$

where Q is a tabulated function[3], and S/N is the signal-to-noise ratio. In general at some range R we let

$$\sqrt{2S/N} \doteq \sqrt{2S_o/N}\cdot e^{u/20m}\cdot (R_p/R)e^{(1/2)\alpha(R_p-R)} \qquad \text{(A-2)}$$

where α is the attenuation coefficient; S_o is the peak in the signal distribution shown in Figure 1, received from the range R_p; $m \doteq \log_{10}e$;

$$u = 10 \log_{10}(S/S_o)\Big|_{R = R_p}. \quad \text{(A-3)}$$

Note that S in Eq. (A-3) is evaluated at R_p, but $\sqrt{2S/N}$ in Eq. (A-2) is evaluated at any R, while S_o is always evaluated at R_p.

We show in Appendix B that the cumulative probability of detection can be obtained approximately as follows: let the intruder appear at some large range R_o, and let the distance $R_o - R_p$ be subdivided into k subintervals of length $\Delta R = 2v_rT_p$, where v_r is the relative velocity (equal to range rate for a collision course) and T_p is the time interval between pulses. Then $\Delta R = 2v_rT_p$ is the distance traveled by the intruder and the protected aircraft during the transmission of two pulses. The probability of not detecting the intruder at all by the time he has reached R_p we denote by $P_o(R_p)$, then the cumulative probability of detection $P_1(R_p)$ is given by $1-P_o(R_p)$. The probability of not detecting in the j^{th} subinterval, for a fixed signal strength, is equal to

$$p_j = 1-Q^2(R_j) , \quad \text{(A-4)}$$

where for simplicity we indicate for the Q-function only the dependence on range, and $R_j = R_o - j\Delta R$. Furthermore,

$$P_o(R_p) = p_k \cdot p_{k-1} \cdots p_1 \cdot p_o \quad \text{(A-5)}$$

where from our definition of ΔR and k we have $R_k = R_p$. Moreover,

$$\ln\left[P_o(R_p)\right] = \sum_{j=0}^{k} \ln p_j = \frac{1}{\Delta R} \sum_{j=0}^{k} \ln p_j \cdot \Delta R \quad \text{(A-6)}$$

We now approximate the sum by an integral,

$$\ln\left[P_o(R_p)\right] \approx \frac{1}{\Delta R} \int_{R_p}^{R_o} dR \, \ln\left[1-Q^2(R)\right] \quad \text{(A-7)}$$

where we also replaced p_j by Eq. (A-4). Then, since $\Delta R = 2v_r T_p$ and $P_1 = 1-P_o$, the cumulative probability of detection is given by

$$P_1(R_p) = 1-\exp\left\{\frac{1}{2v_r T_p}\int_{R_p}^{R_o} dR \ln\left[1-Q^2(R)\right]\right\} . \qquad (A-8)$$

In practice any large range will serve as R_o as long as $Q^2(R_o) << Q^2(R_p)$, but one should not extend R_o to infinity, since one will obtain an infinite number of false alarms in the infinite time interval required to travel that infinite distance.

The average cumulative probability of detection $\bar{P}_1$ is given by

$$\bar{P}_1 (R_p) = \int_{-\infty}^{\infty} du\ p(u)\ P_1(R_p) \qquad (A-9)$$

where the probability density function $p(u)$ for the signal strength is given by Eq.(1). $P_1(R_p)$ in the integrand of Eq. (A-9) is a function of u because Q^2 is a function of $\sqrt{2S/N}$, see Eq. (A-1), and $\sqrt{2S/N}$ in turn is a function of u, see Eq. (A-2).

We now require that $\bar{P}_1(R_p) = 0.95$ when $v_r = 582$ knots, and solve for S_o/N. The solution was obtained numerically by interpolation for $P_{fa} = 10^{-5}$, $2T_p = 1$ second, $R_p = 6080$ and 14,740 feet, and the two signal strength density functions shown in Figure 1. The calculated S_o/N are presented in Table 1.

Appendix B. Probability of Occurence of Two Consecutive Successes.

Consider a sequence of events, where an event may be the reception of a pulse, or the drawing of a ball from an urn containing a mixture of red and black balls, etc. We wish to find the probability that in a sequence of n events success will occur at least twice consecutively; by success we mean that the signal strength of the pulse will exceed a threshold, or the drawn ball will be red, etc.

Let Q be the constant probability of success during any one of the events. Let P_n be the probability of not obtaining two consecutive successes in n events, so that our desired probability of two consecutive successes is equal to $1-P_n$.

P_n can be related to P_{n-1} and P_{n-2}: a) if there was no success in the n^{th} event, then it does not matter whether the sequence of n-1 previous events ended with a success or failure, as long as one did not obtain two consecutive pulses* during those n-1 events; thus one of the terms contained in P_n is $(1-Q)P_{n-1}$; b) if there was a success in the n^{th} event, then there must have been a failure in the $n-1^{st}$ event, and no two consecutive pulses* during the previous n-2 events; thus the second term contained in P_n is $Q(1-Q)P_{n-2}$, and

$$P_n = (1-Q)P_{n-1} + Q(1-Q)P_{n-2}. \qquad \text{(B-1)}$$

In particular, for the first few n:

$$P_0 = 1$$

$$P_1 = 1$$

$$P_2 = 1-Q^2$$

$$P_3 = 1-2Q^2 + Q^3$$

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* I.e., successes.

$P_4 = 1-3Q^2 + 2Q^3$

$P_5 = (1-Q)^2(1 + 2Q-Q^2-Q^3)$

$P_6 = (1-Q)^3(1 + 3Q + Q^2-Q^3)$

Let us now define

$$P_{2m} = (1-Q)^m Q^m F_m \tag{B-2}$$

$$P_{2m+1} = (1-Q)^m Q^{m+1} G_m \tag{B-3}$$

Substitution into Eq. (B-1) yields two equations (for $n = 2m$ and $n = 2m + 1$) which can be manipulated to obtain an expression for the G_m's in terms of F_m's,

$$G_m = F_{m+1} - F_m, \tag{B-4}$$

and we also obtain a recurrence relation for F_m's,

$$F_{m+1} = F_m \cdot (1 + Q)/Q - F_{m-1}. \tag{B-5}$$

this is the same recurrence relation as the one satisfied by Tchebyscheff polynomials U_m.[6] We thus obtain:

$$F_m = U_m\left(\frac{1+Q}{2Q}\right) \tag{B-6}$$

$$G_m = U_{m+1}\left(\frac{1+Q}{2Q}\right) - U_m\left(\frac{1+Q}{2Q}\right) \tag{B-7}$$

Note that usually $U_m(x)$ is utilized only for $|x| \leq 1$; however, since the polynomial contains only a finite numberof terms, there is no reason why one should not also use the same polynomial for $|x| > 1$, which is the case here.

We have[7]

$$U_m(x) = (2x)^m \sum_{k=0}^{\leq (1/2)m} (-1)^k \left[\frac{(m-k)!}{(m-2k)!\,k!}\right](2x)^{-2k} \tag{B-8}$$

For large 2x the dominant term in Eq. (B-8) is the first term in the summation. If we approximate the sum by the first term, substitution in Eqs. (B-2) and (B-3) yields

$$P_{2m} \approx (1-Q^2)^m \tag{B-9}$$

and

$$P_{2m+1} \approx (1-Q^2)^m. \tag{B-10}$$

Note that we would have obtained the same result if we had partitioned our sequence of single events into a sequence of double events with half as many terms and a probability of success given by Q^2 for the double event. The difference between this double event problem and the actual problem is that in the double event problem the sequence "failure-success-success-failure" is regarded as two double events, neither one yielding two successes, while in the actual problem the above sequence does yield the two desired consecutive successes. Figure 11 shows the exact and approximate probability for six events. The approximation is a conservative underestimate of the true probability.

In the above discussion we have assumed that Q is constant. However, in the actual situation discussed in Appendix A the Q-function changes with range as the intruder approaches. Since the probability of detection increases from practically zero to almost one in a rather narrow range (see Figures 2-9), only a small number of pulses will contribute to the cumulative probability of detection, thus the results shown in Figure 11 will be typical, and we have assumed that the above conservative approximation can be used, especially because we are

mostly interested in the relative performance of different systems, and we use the same approximation for all systems. Since the approximation is an underestimate, the true cumulative probability of detection will exceed 95% at R_p for the worst case.

Appendix C. Alarm Rates

We are considering intruders on linear paths with random headings in the horizontal plane. Let us select all the intruders with a particular heading and speed so that the protected aircraft velocity vector and the velocity vectors of all those intruders form the same relative velocity vector. We can assume that the intruders are stationary and the protected aircraft moves with the relative velocity v_r. Let w be the coordinate normal to the relative velocity vector and in the horizontal plane, see Figure 12. Let $P(w)$ be the probability that an intruder is detected at least once during a long time interval Δt (where $v_r \cdot \Delta t$ is much greater than a range at which the probability of detection is significant), if the closest distance between the two aircraft at any time is w (this happens when $r = 0$ in Figure 12). A segment of length dw (between w and $w + dw$) during Δt sweeps out an area $dwv_r \cdot \Delta t$. Let n be the number of intruders per unit area (with this particular heading and speed). Then the number of intruders detected during Δt on this strip of width dw will be $ndwv_r \cdot \Delta t P(w)$. The average rate of detection N is obtained by dividing by Δt and integrating over w:

$$N = 2nv_r \int_0^{w_0} P(w)dw, \qquad \text{(C-1)}$$

where w_0 is the maximum range of detection, analogous to R_0 in Appendix A.

To obtain $P(w)$ we proceed as in Appendix A. The cumulative probability of detection $P(w)$ is given by $1-P_f(w)$, where $P_f(w)$ is the probability of failing to detect at all. $P_f(w)$ is given by an equation similar to Eq. (A-5), but the product extends over intruder positions from $+r_m$ to $-r_m$, where r_m is the maximum r-coordinate for detection, analogous to R_0 in Appendix A. Furthermore, since the aircraft are not on a collision course, relative bearing does not remain constant. Let us assume that the signal strength fluctuations due to the fluctuations in the antenna

pattern occur much more rapidly than changes in signal strength due to changes in distance as the protected aircraft proceeds with velocity v_r. Then we must integrate over the signal strength distribution first, and then sum over the ranges as in Appendix A, yielding approximately

$$P(w) = 1-\exp\left\{ \frac{1}{2v_r T_p} \int_{-r_m}^{r_m} dr \int_{-\infty}^{\infty} du\ p(u)\ \ln(1-Q^2) \right\} \qquad \text{(C-2)}$$

where Q depends on u and R as shown in Eqs. (A-1) and (A-2), and

$$R = \sqrt{w^2 + r^2}, \qquad \text{(C-3)}$$

see Figure 12.

The integrand in Eq. (C-2) is such that we could interchange the integrations over u and r, i.e., we could perform the integration over r first, and still obtain the same result. This means that our assumption above that antenna pattern fluctuations yield more rapid signal strength changes than changes in range is not vital, the opposite assumption would give the same result.

Figures 13 and 14 show P(w) vs. w for the two design ranges, two signal strength distributions, and two types of radiation. We used the median relative velocity of Table II in Reference 1 in these calculations. However, P(w) is not a very sensitive function of v_r, see Figure 15, where P(w) for w = 100,000 feet is shown vs. $2T_p$ for the minimum, median, and maximum relative velocity of Table II, Reference 1. Consequently, to obtain N (and M) we used the median v_r = 192 knots instead of averaging over the relative velocity distribution.

If we compare Figures 13 and 14, obviously the microwave systems are inferior to the millimeter wave systems, since detection for microwave PWI extends needlessly far, and, therefore, many unnecessary alarms are generated.

To obtain M, the number of alarms generated by GA aircraft at an AC aircraft during a flight of 800 seconds in a typical terminal area, we simplify the actual situation by assuming that in the typical encounter we always obtain the median $v_r = 192$ knots, since a) P(w) does not depend very much on v_r, see Figure 15, and b) we are mainly interested in relative numbers to compare different systems, and the relative results are not very much affected by such simplifying assumptions. We are also ignoring any corrections to range due to altitude differences, because most detection ranges exceed greatly the possible altitude differences; i.e., for a fixed limit to detection, the volume containing detected intruders would be a part of a sphere with the protected aircraft at the center, and the GA intruders contained in the slice from 1000 to 10,000 feet altitude, and we are replacing the curved boundary of the sphere by the straight boundary of a cylinder.

Then

$$M \approx (N/n)n_o t_o = 2n_o v_r t_o \int_0^{w_o} dw P(w), \qquad (C-4)$$

where $t_o = 800$ seconds, and $n_o = 0.0270$ per $(n.mi.)^2$ for future operations,* $n_o = 0.00636$ per $(n.mi.)^2$ for current operations,* n_o being the total equivalent area density of intruders for all speeds, same as in Reference 1. By "equivalent area density" we mean the following: the actual aircraft are distributed at random over the whole volume of terminal airspace; if we pick an area in the horizontal plane of one $(n.mi.)^2$ and calculate the number of aircraft in a cylinder above and below this area, then this number is n_o.

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* For intruders within an altitude layer of ±800 feet; when there is no altitude discrimination, we assume that the area density is increased by the ratio of layer thicknesses (10,000-1000)/1600.

Appendix D. Warning Times

We are interested in two quantities: a) the average warning time $\bar{T}_w$; b) the probability that the warning time may be less than 15 seconds, which in Reference 1 was assumed to be the minimum warning time required.

In Appendix A we defined p_j to be the probability of failure to detect the intruder in the j^{th} range interval, see Eq. (A-4). $P_o(R_j)$ is the cumulative probability of failure to detect as the intruder approaches from some great range R_o to the range R_j. Then the probability of detecting the intruder first in the n^{th} range interval is equal to

$$\begin{aligned} P_F(R_n) &= (1-p_n)P_o(R_{n-1}) \\ &= (1-p_n)p_{n-1}\cdot p_{n-2}\cdots p_1\cdot p_o \end{aligned} \tag{D-1}$$

and

$$\ln\left[P_F(R_n)/(1-p_n)\right] = \sum_{m=0}^{n-1} \ln p_m \quad . \tag{D-2}$$

Again we approximate the sum by an integral, consequently,

$$P_F(R_n) \approx Q^2(R_n)\exp\left\{\frac{1}{2v_rT_p}\int_{R_n}^{R_o} dR \ln\left[1-Q^2(R)\right]\right\} \tag{D-3}$$

Then for a particular value of signal strength in the distribution shown in Figure 1 the expected value of the warning time when the two aircraft are on a collision course is given by

$$T_w = \sum_n (R_n/v_r)P_F(R_n)/\sum_n P_F(R_n) \tag{D-4}$$

where for a collision course the range rate is given by v_r (the relative velocity), and the sums extend over all ranges. We again approximate the sums by integrals,

$$T_w \approx \left[\frac{1}{v_r} \int_0^{R_o} dR \; R \; P_F(R) \right] \Big/ \int_0^{R_o} dR \; P_F(R). \qquad \text{(D-5)}$$

The average warning time $\bar{T}_w$ is obtained by averaging over the signal strength distribution,

$$\bar{T}_w = \int_{-\infty}^{\infty} du \; p(u) T_w . \qquad \text{(D-6)}$$

For the numerical calculations presented in Table I we let $v_r = 192$ knots, the median relative velocity of Table II in Reference 1.

Note that Eq. (D-5) could be written as

$$T_w = R_F / v_r \qquad \text{(D-7)}$$

where R_F would be the ratio of the two integrals, and physically R_F would be the expected range of first detection for a particular signal strength.

Let us now consider the probability that the warning time may be less than $T_m = 15$ seconds. This may happen because for any signal level there is a non-vanishing probability that the range of first detection will be less than $v_r T_m$. Consequently, we have to consider: a) the probability density function for range of first detection for a fixed signal strength; b) the signal strength distribution; c) the relative velocity distribution.

Let us study the probability density function for the range of first detection at a fixed signal strength. If one examines Eq. (D-3) and the manner in which the Q function depends on R_p and S_o/N (see Eqs. (A-1) and (A-2)), it becomes obvious that for the same type of radiation (microwaves or millimeter waves) one will obtain the same P_F if the combinations of parameters

$$\eta = \sqrt{2S_o/N} \; e^{u/20m} \; R_p \; e^{(1/2)\alpha R_p} \qquad \text{(D-8)}$$

and

$\Delta R = 2v_r T_p$ (D-9)

have the same value, e.g., one can increase $\sqrt{2S_o/N}$ and decrease R_p so as to keep η constant, and obtain a constant P_F. For example, for millimeter waves $\eta = 10^6$ feet may be regarded as: a) $S_o/N + 13.1$ db for $R_F = 6080$ feet, $\sigma_+ = 2.5$ db, $\sigma_- = 5$db, $S_o/N = 21.4$ db; b) $S_o/N + 4.3$ db for $R_p = 6080$ feet, $\sigma_+ = 5$db, $\sigma_- = 10$ db, $S_o/N = 30.2$ db; c) $S_o/N - 3.9$ db for $R_p = 14,740$ feet, $\sigma_+ = 2.5$ db, $\sigma_- = 5$db, $S_o/N = 21.0$ db; and d) $S_o/N = 12.6$ db for $R_p = 14,740$ feet, $\sigma_+ = 5$db, $\sigma_- = 10$ db, $S_o/N = 29.7$ db. Figure 16 shows the probability $P_F(R)$ of detecting at range R first for a typical case of $\eta = 10^6$ feet and $\Delta R = 324$ feet (which corresponds to $v_r = 192$ knots and $T_p = 1/2$ second). Note that 95% of the first detections occur within less than $\pm 10\%$ of the mean range R_F. Consequently, we simplified our calculations by assuming that for a fixed signal strength all first detections occur at the mean range R_F.

We now have to consider the effects of the signal strength distribution and the relative velocity distribution on the warning time T_w. For a particular R_F (corresponding to some particular signal strength) we can find the relative velocity V which will give just 15 seconds warning, and then obtain from Figure 10 the cumulative probability $1-\Phi_{v_r}(V)$ that the relative velocity will exceed this $V = R_F/T_m$. If we integrate $1-\Phi_{v_r}(V)$ over the signal strength, we obtain the desired probability Φ that the warning time will be less than $T_m = 15$ seconds,

$$\Phi = \int_{-\infty}^{\infty} du\, p(u) \left[1 - \Phi_{v_r}(V) \right] . \qquad \text{(D-10)}$$

Of course, for some signal strengths R_F will be so large that the required velocity V will not occur, i.e., $1-\Phi_{v_r}(V) = 0$.

Eq. (D-10) was evaluated by the use of numerical methods. One as to go through the intermediate calculation of R_F from Eqs. (D-7) and (D-5) to find the relative velocity $V = R_F/T_m$ for each particular u in the integrand of Eq. (D-10).

Figure 10 was computed numerically from the data for AC and GA aircraft speeds in a typical terminal area[5], taking into account the random distribution of relative headings. Let v_1 be the speed of the AC, v_2 the speed of the GA aircraft. Let the relative heading be θ, then the relative velocity is given by

$$v_r^2 = v_1^2 + v_2^2 + 2v_1v_2\cos\theta , \qquad \text{(D-11)}$$

see Eq. (A-2) and Figure 2 in Reference 1.

Let $p_i(v_i)dv_i$, $i = 1$ or 2, be the probability that v_i will have a value between v_i and $v_i + dv_i$. The probability that θ will have a value between θ and $\theta + d\theta$ is $d\theta/2\pi$. Then

$$1 - \Phi_{v_r}(V) = \iiint_{v_r > V} p_1(v_1)p_2(v_2)dv_1dv_2d\theta/2\pi \qquad \text{(D-12)}$$

where the integration is performed over those values of v_1, v_2, and θ which yield $v_r > V$. If we substitute V for v_r in Eq. (D-11), we can solve for the limiting angle θ_L at which $v_r = V$ for a specific v_1 and v_2,

$$\cos\theta_L = (V^2 - v_1^2 - v_2^2)/2v_1v_2 . \qquad \text{(D-13)}$$

Of course, for some v_1 and v_2 we obtain $v_r > V$ for all angles θ, in which case we should let $\theta_L = \pi$. We can then perform the integration over θ in Eq. (D-12), which yields

$$1 - \Phi_{v_r}(V) = \int dv_1 \int_{v_r > V} dv_2 \, (\theta_L/\pi) p_1(v_1)p_2(v_2) \qquad \text{(D-14)}$$

The rest of the integrations in Eq. (D-14) were performed numerically.

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2. J. I. Marcum, A Statistical Theory of Target Detection by Pulsed Radar, Rand Corp. report RM-754(1947); Mathematical Appendix, RM-753(1948); see also M.I. Skolnik, Introduction to Radar Systems (McGraw-Hill Book Co., Inc., New York, 1962), Section 2.5.

3. J. I. Marcum, Table of Q Functions, Rand Corp. report RM-339(1950).

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5. C. Becklein, Line-of-Sight Model for Terminal Traffic, Control Data Corp. report (June 1970), Figure 3.

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7. V. Mangulis, Handbook of Series for Scientists and Engineers (Academic Press, New York, 1965), p. 49, Eq. (85).

* Second report in this volume.

** First report in this volume.

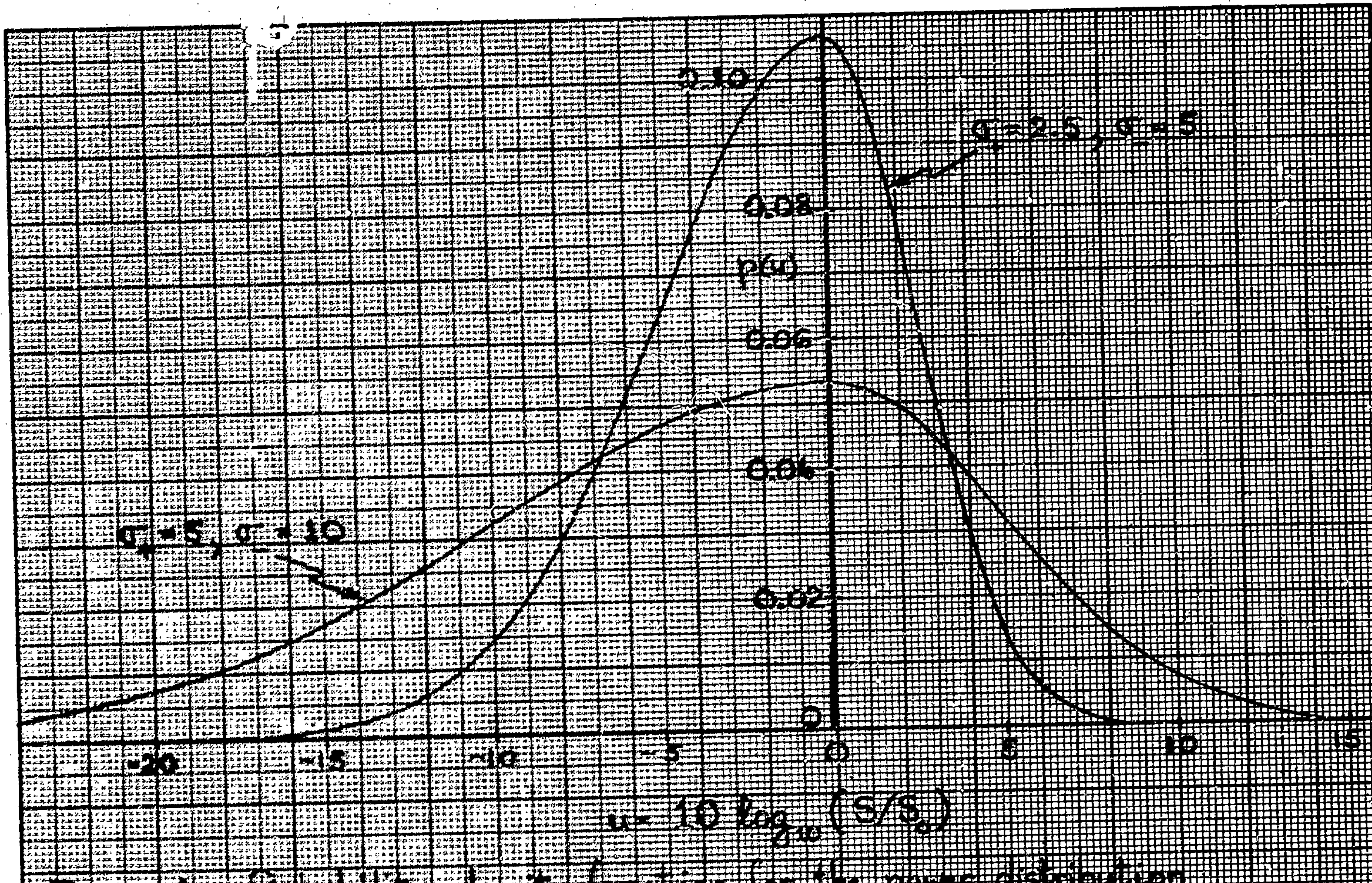


Figure 1. Probability density function for the power distribution

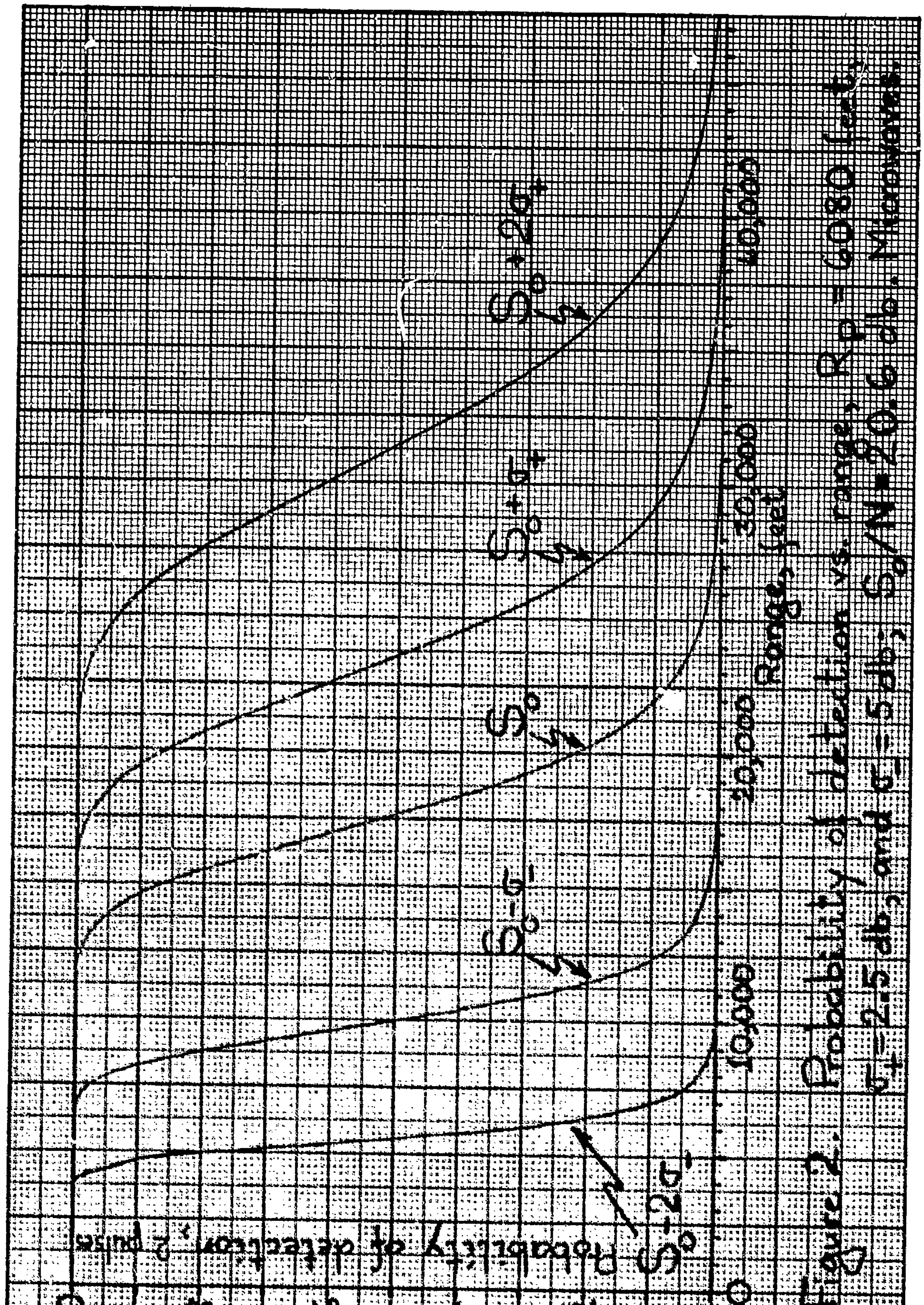


Figure 2. Probability of detection vs. range, R.P. = 6080 feet, $\sigma_+ = 2.5$ db, and $\sigma_- = 5$ db; $S_o/N = 20.6$ db. Microwaves.

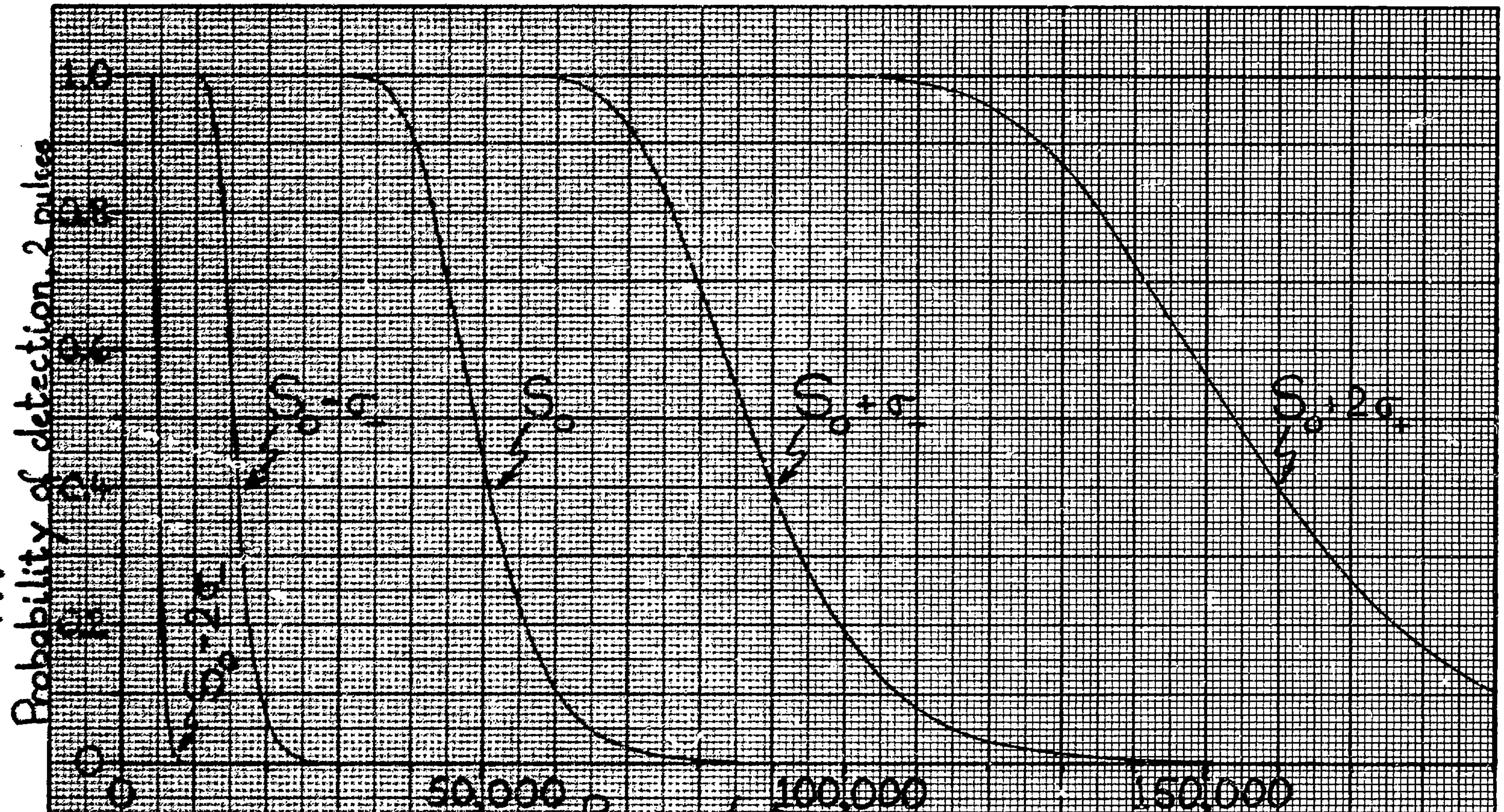


Figure 3. Probability of detection vs. range, $R_p = 6080$ feet, $\sigma_+ = 5$ db, and $\sigma_- = 10$ db; $S_o/N = 29.4$ db at R_p. Microwaves.

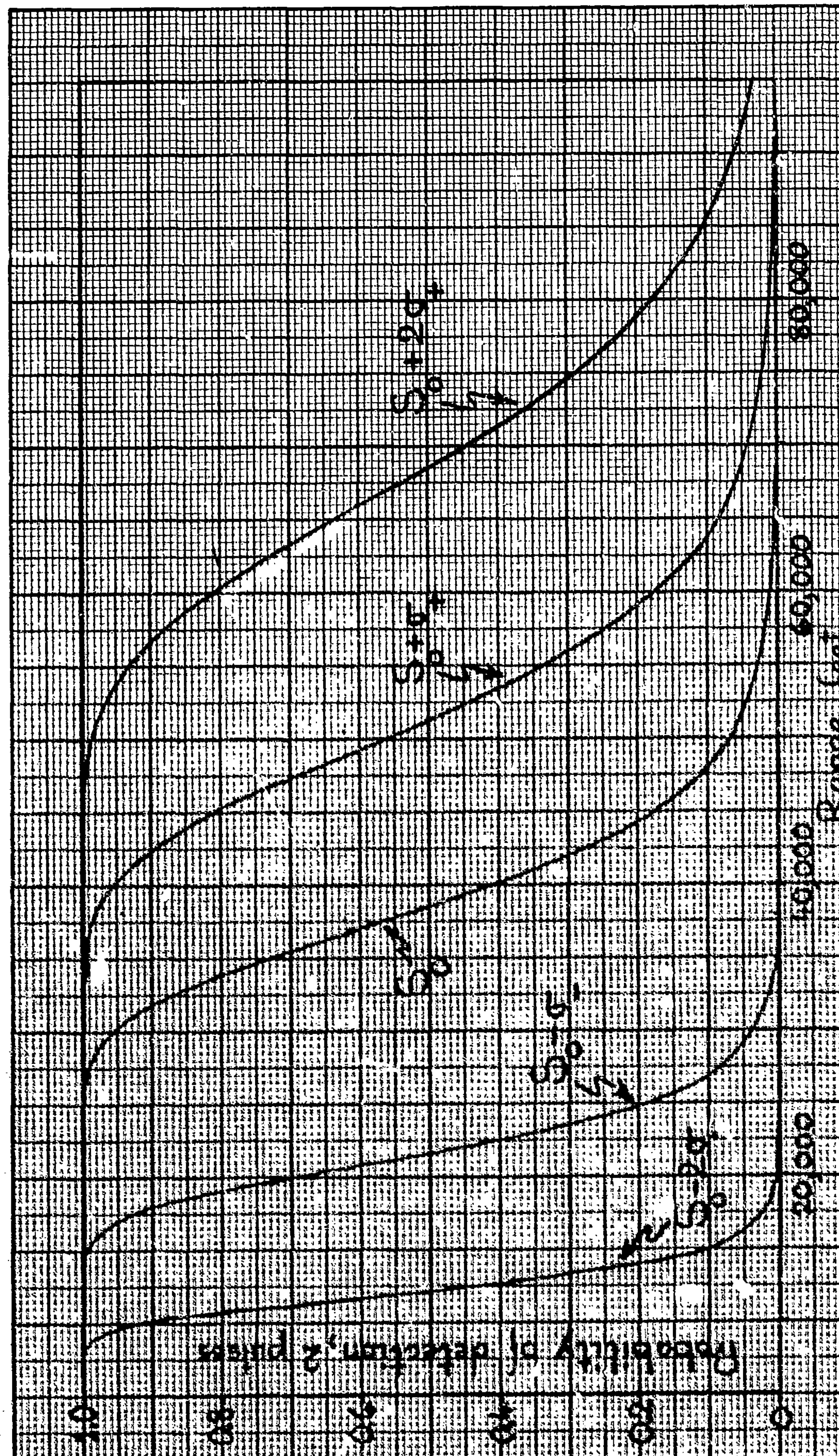


Figure 4. Probability of detection vs. range, $R_p = 14,740$ feet, $\sigma_+ = 2.5$ db, and $\sigma_- = 5.0$ db; $S_o/N = 19.7$ db at R_p. Microwaves.

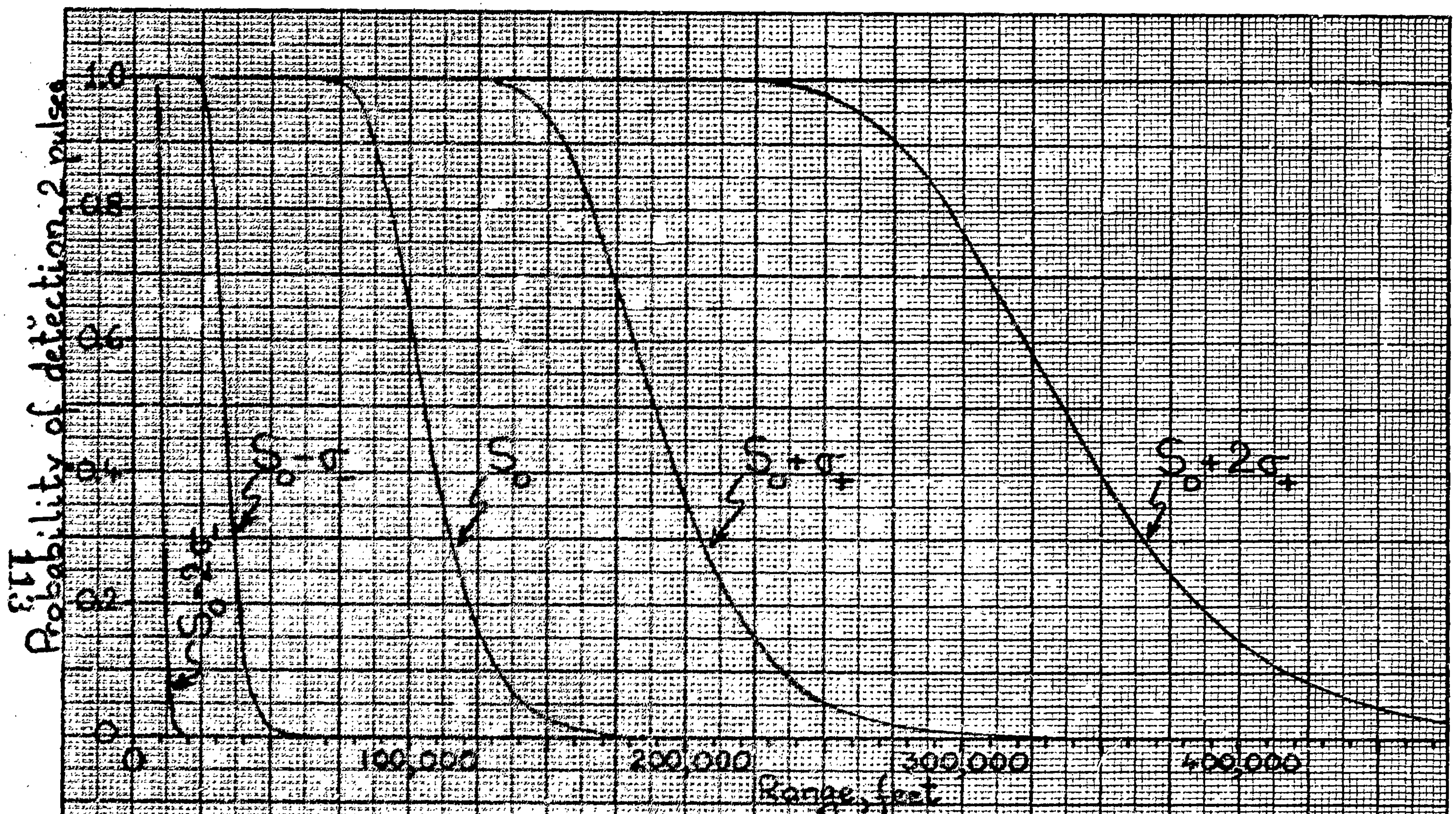


Figure 5. Probability of detection vs. range, $R_p = 14,740$ feet, $\sigma_+ = 5$ db, and $\sigma_- = 10$ db; $S_p/N = 28.5$ db at R_p. Microwaves.

113

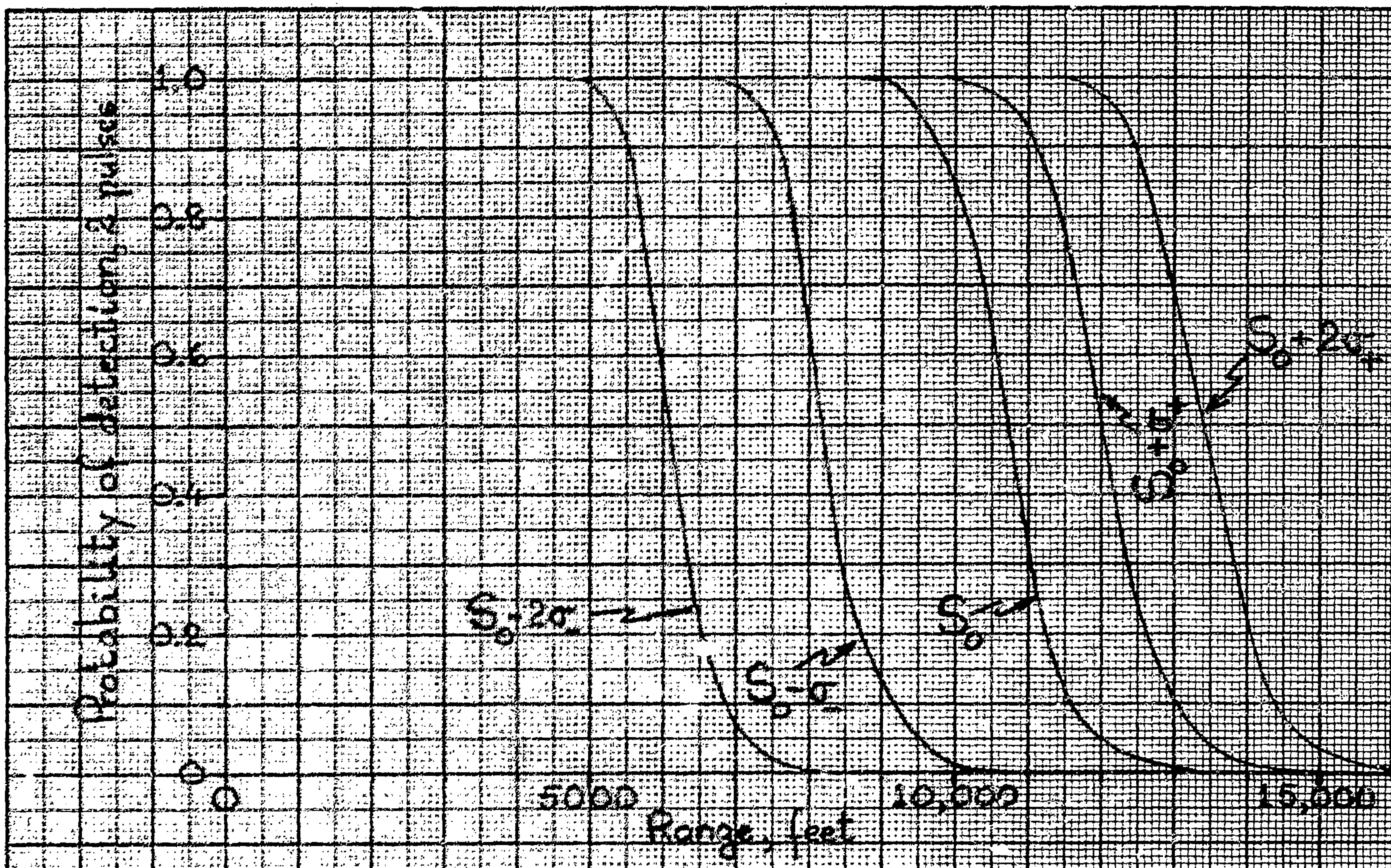


Figure 6. Probability of detection vs. range, $R_p = 6080$ feet, $\sigma_+ = 2.5$ db, and $\sigma_- = 5$ db; $S_o/N = 21.4$ db. Millimeter waves.

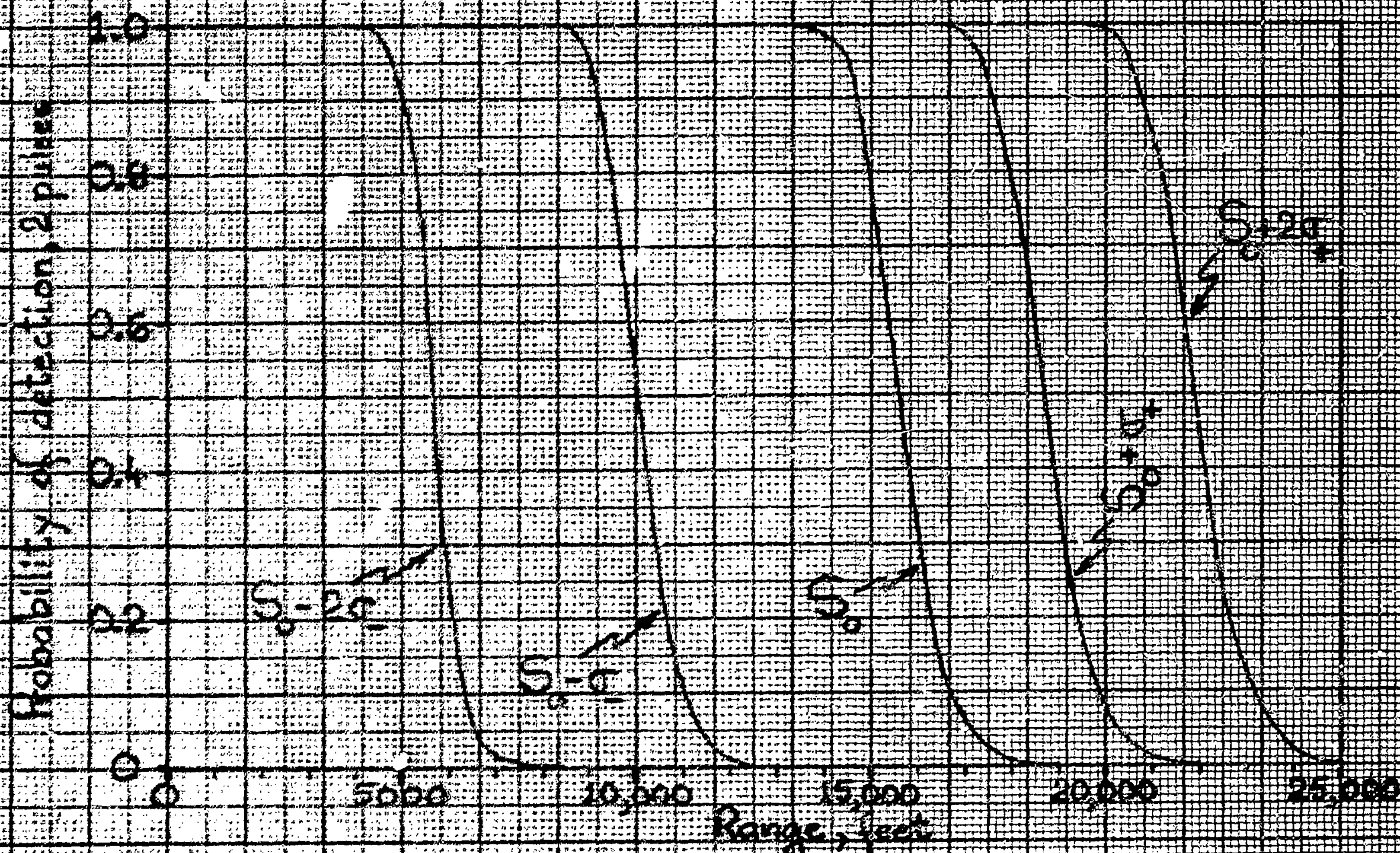


Figure 7. Probability of detection vs. range, $R_p = 6080$ feet, $\sigma_+ = 5$ db, and $\sigma_- = 10$ db; $S_o/N = 30.2$ db. Millimeter waves.

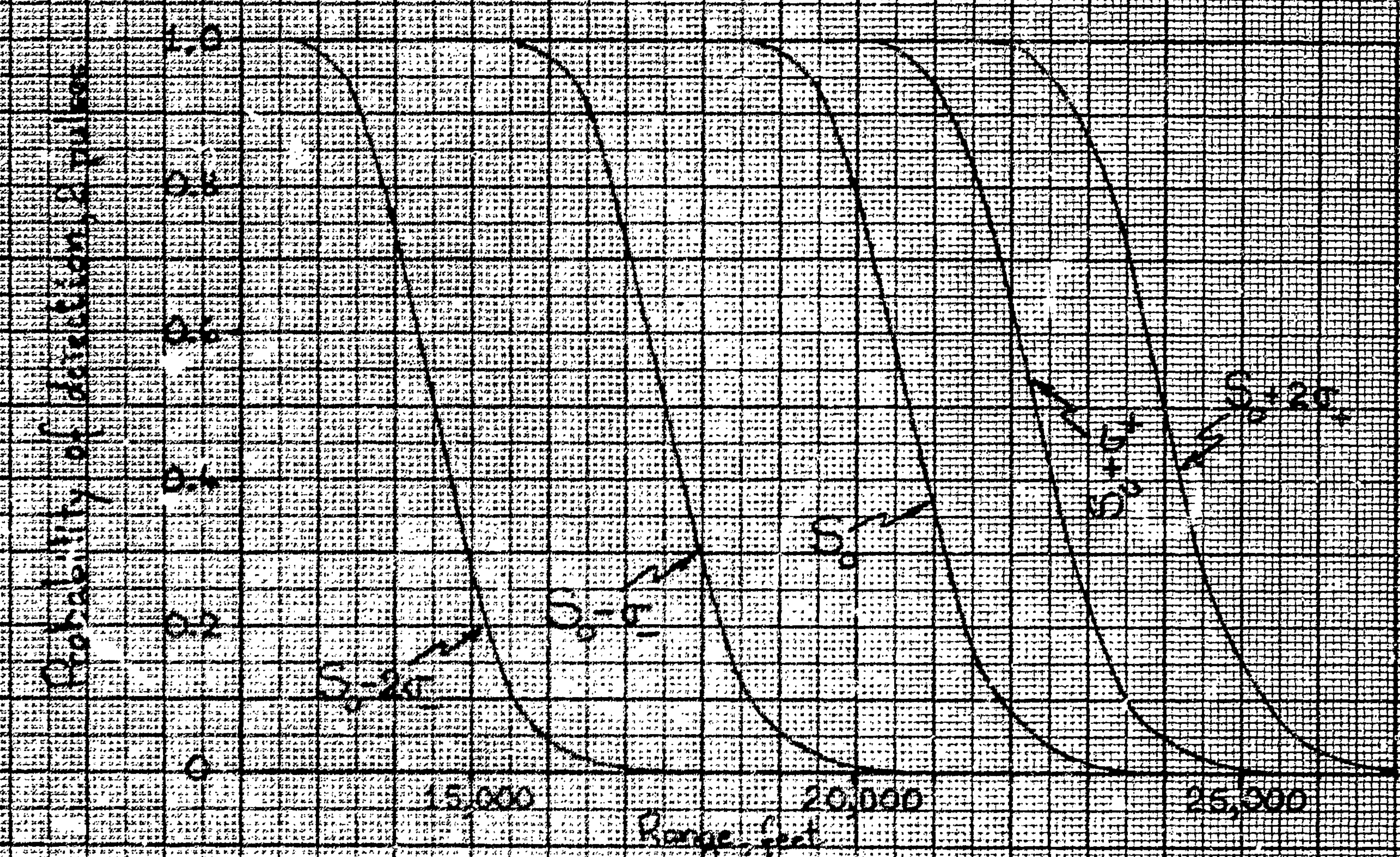


Figure 8. Probability of detection vs. range, R_p = 14,740 feet, σ_+ = 2.5 db, and σ_- = 5 db; S_o/N = 21.0 db. Millimeter waves.

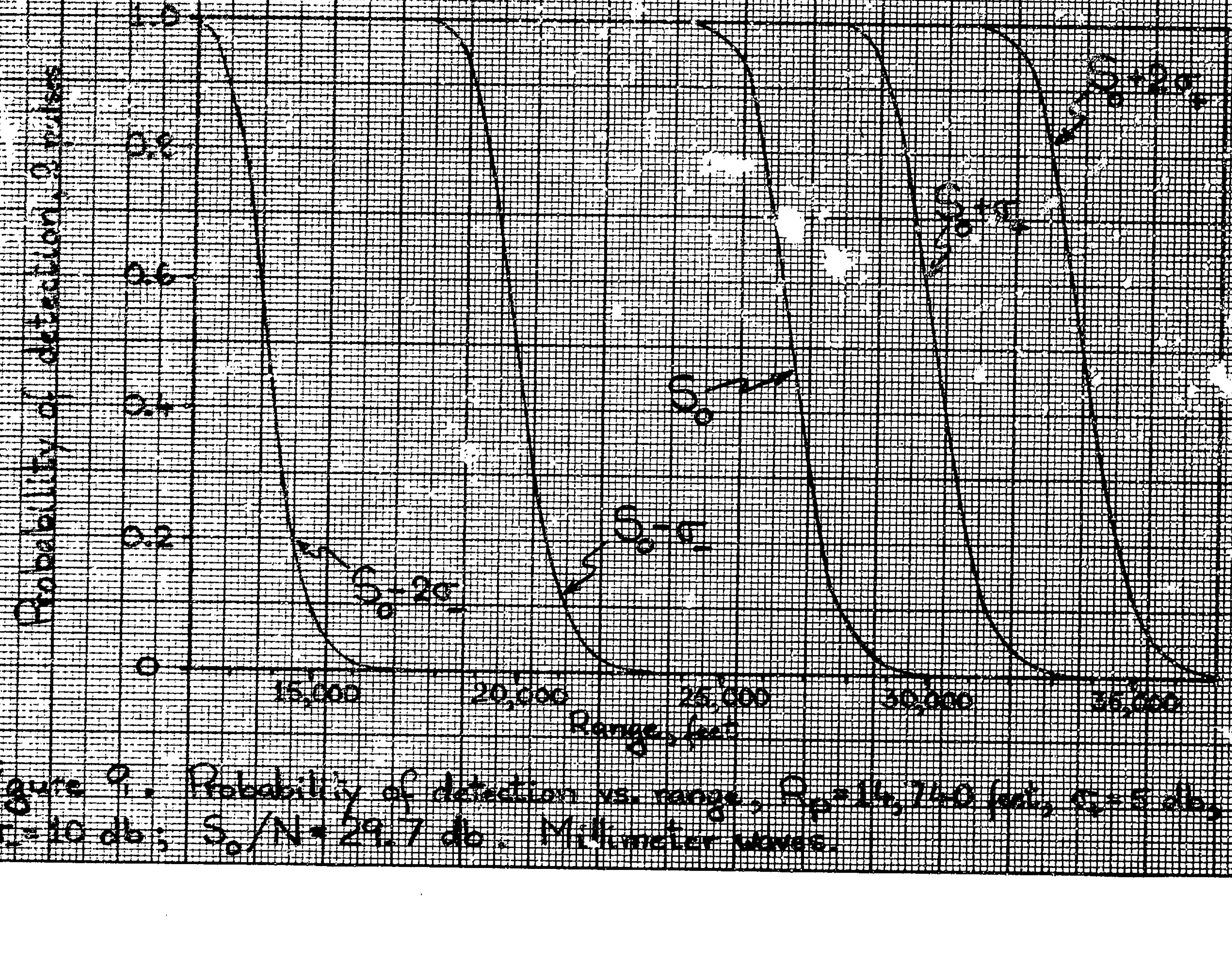
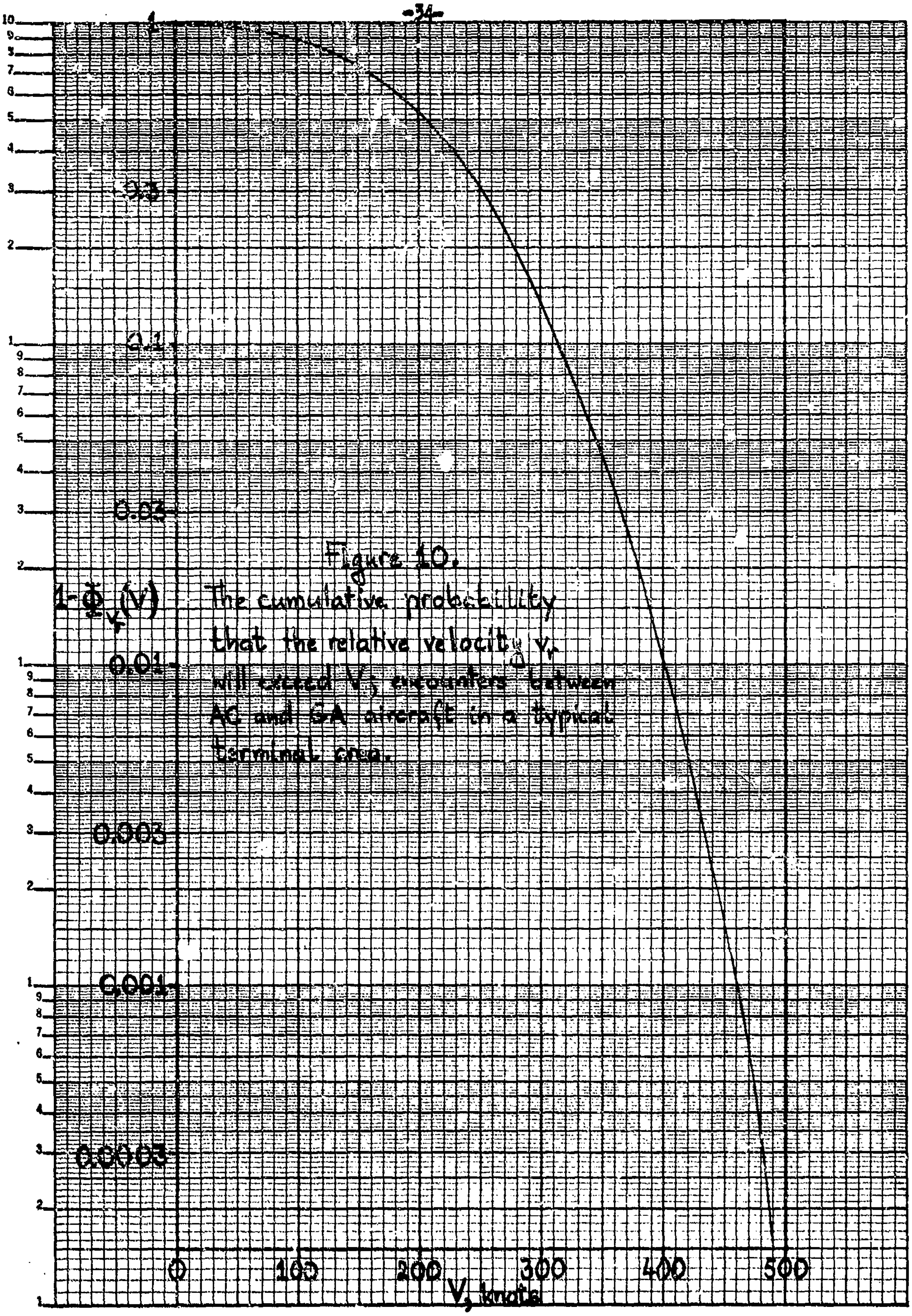


Figure 9. Probability of detection vs. range, R_p = 14,740 feet, σ_+ = 5 db, and σ_- = 10 db; S_o/N = 29.7 db. Millimeter waves.



1-Φ_v(V)
Figure 10.
The cumulative probability that the relative velocity v_r will exceed V; encounters between AC and GA aircraft in a typical terminal area.
1
0.3
0.1
0.03
0.01
0.003
0.001
0.0003
0
100
200
300
400
500
V, knots

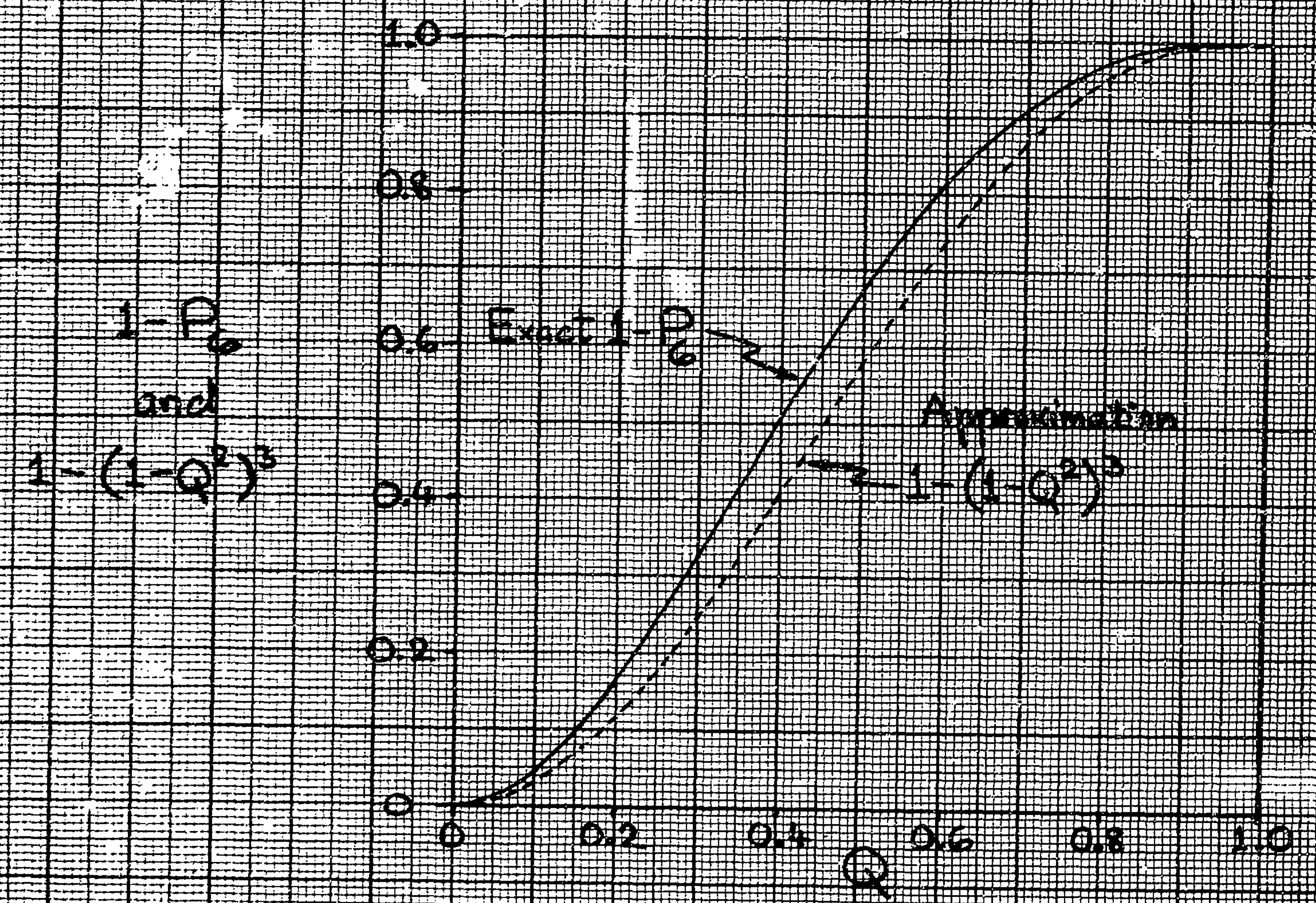


Figure 11. Probability of two consecutive successes in six trials vs. probability of success in a single trial.

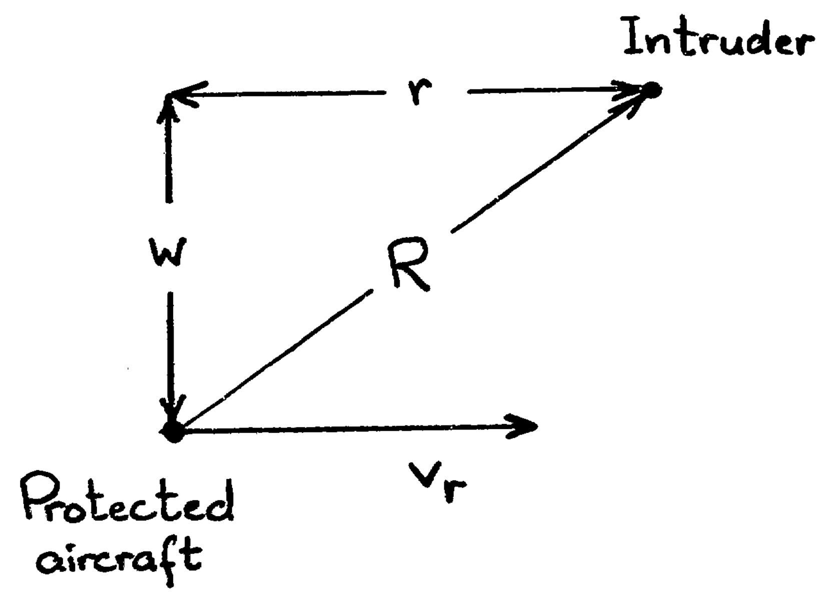


Figure 12. Encounter geometry

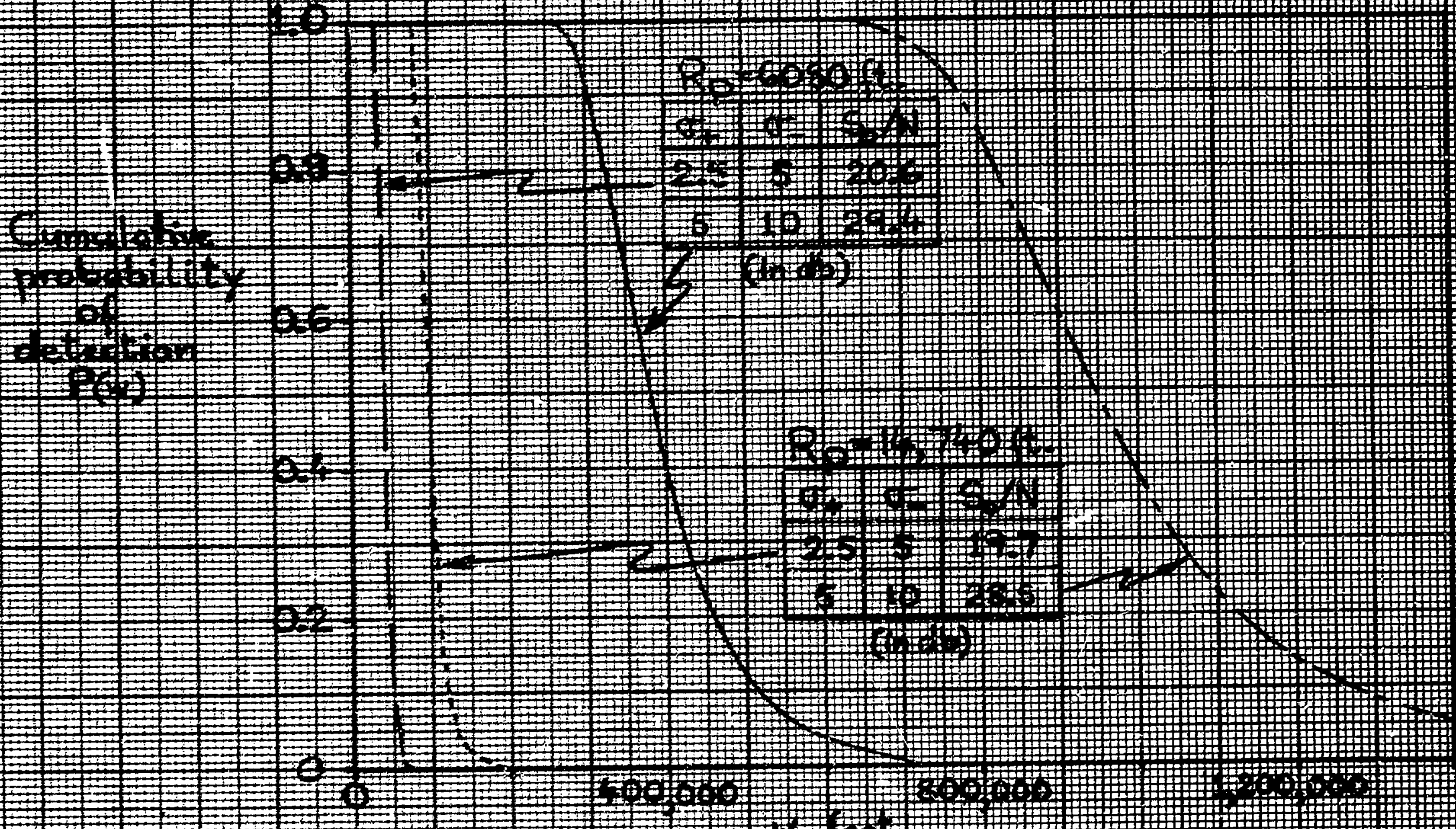


Figure 13. Cumulative probability of detection vs. distance of closest approach. Microwaves. $v_r = 192$ knots, $T_p = 1/2$ second.

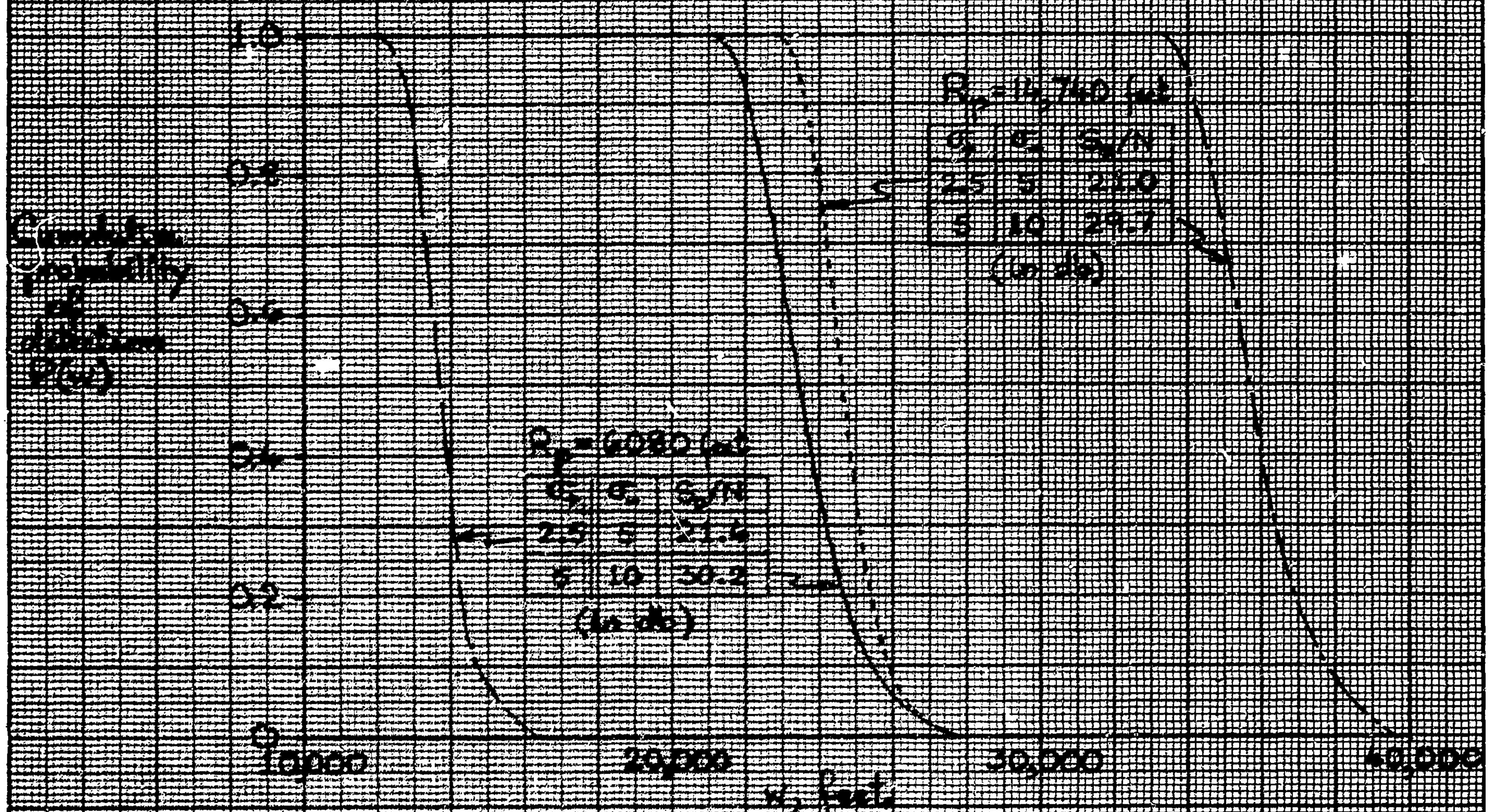


Figure 18. Cumulative probability of detection vs. distance at closest approach. Multimeter waves, attenuation 6.8 db/n.mi. v_p = 192 knots, T_p = 1/2 sec.

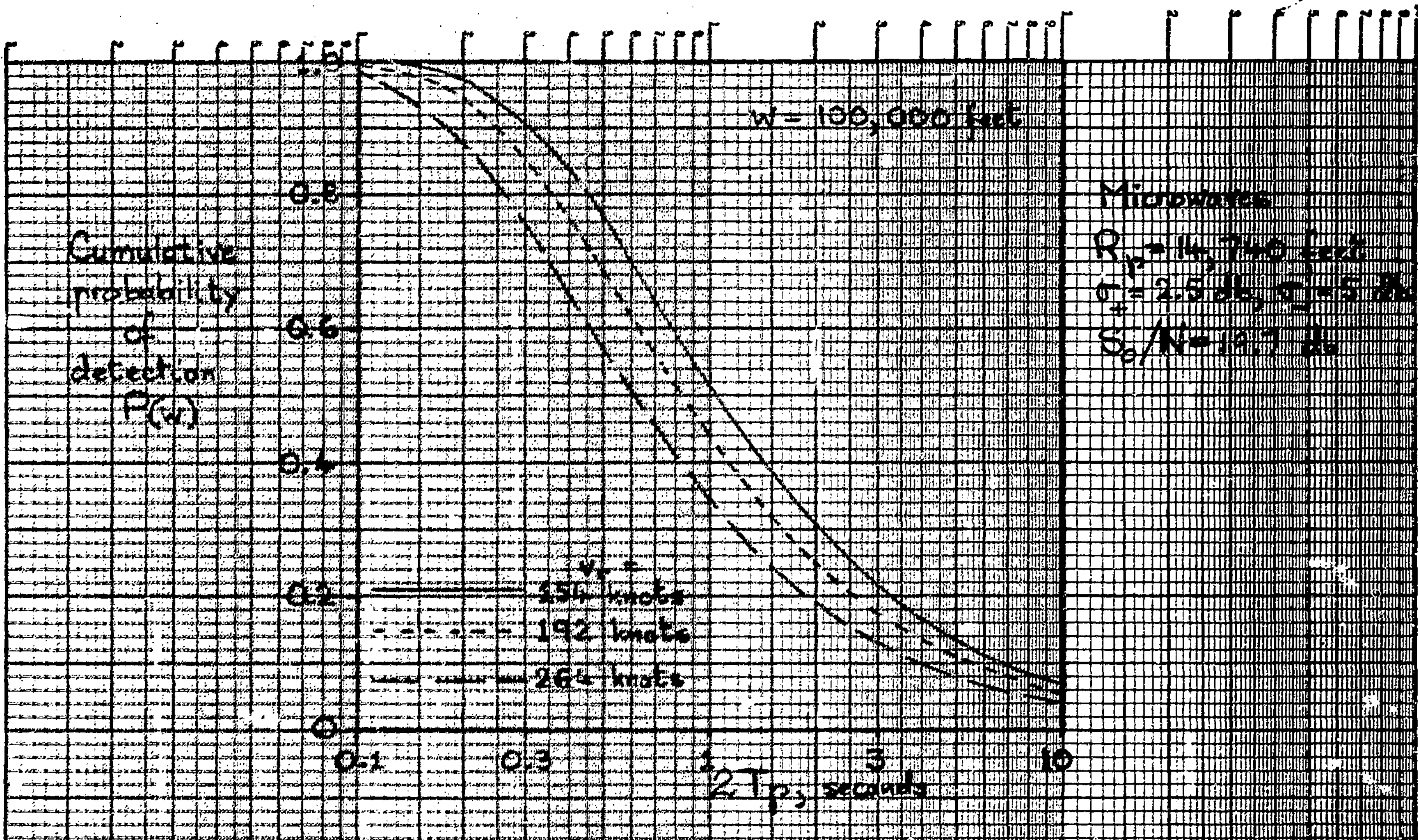


Figure 15. Cumulative probability of detection vs. 2/(pulse rate).

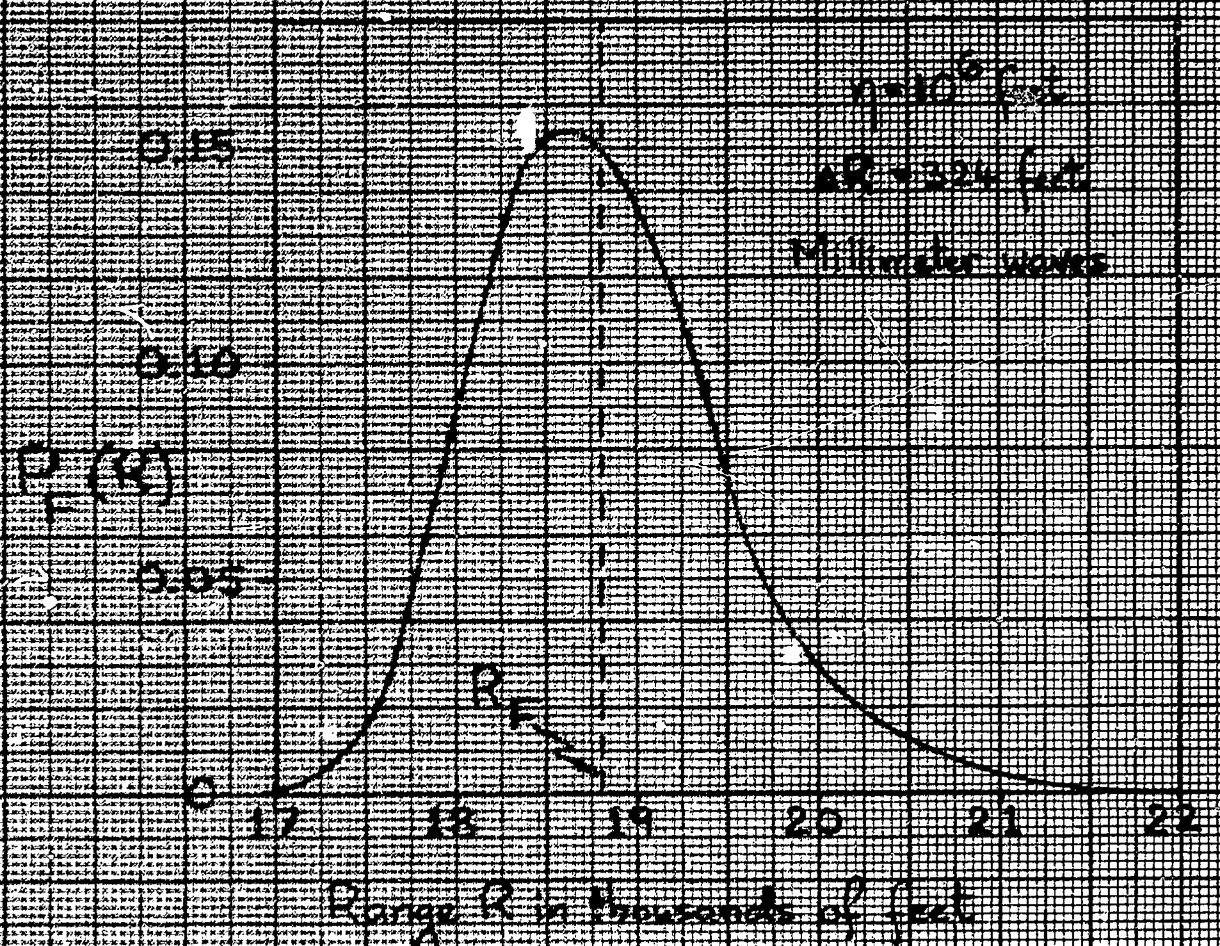


Figure 16. Probability that detection will occur first at range R.